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A local stability condition for dc grids with constant power loads ⋆

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Abstract: Currently, there are an increasing number of power electronics converters in electrical grids, performing the most diverse tasks, but most of them, work as constant power loads (CPLs). This work presents a sufficient condition for the local stability of dc linear time-invariant circuits with constant power loads for all the possible equilibria (depending on the drained power) of the systems. The condition is shown as a method with successive steps that should be met. Its main step is expressed as a linear matrix inequality test which is important for easiness of verification reasons. The method is illustrated with two examples: a single-port RLC circuit connected to a CPL and a two-port linear dc circuit connected to two CPLs.

Keywords: Local stability, dc LTI circuits, constant power loads, frequency domain methods, linear matrix inequalities.

1. INTRODUCTION

The current trend in power grids is to increase the presence of electronic power converters due to their versatility to transform, condition and accumulate electrical energy. However, most of these devices behave as constant power loads. This type of non-linear loads can compromise the stability of the electrical system since, incrementally, they behave as negative resistive elements. So, this characteristic puts at risk the quality of the supply and the integrity of the electrical system. For these reasons, the development of new and, if possible, easy to check conditions for the stability of grids with CPLs is required.

The stability of electrical circuits with constant power loads connected to them has been studied using different approaches in the literature, i.e. Middlebrook (1976), Belkhayat et al. (1995b), Belkhayat et al. (1995a), Sanchez et al. (2014). See, also, the recent survey by Singh et al. (2017) for a state of the art in the behaviour and typical effects of CPLs, stability criteria and compensation techniques.

In this work, we present a new frequency domain method applied to the linearization of dc grids feeding CPLs around the equilibrium. This method is based on the properties of negative imaginary (NI) systems, e.g. Petersen and Lanzon (2010), and it establishes a sufficient condition for the stability of the system for all the possible equilibria that come from sweeping the power of the CPLs. To illustrate the method we apply it to two examples: one port and a two port linear time invariant (LTI) dc electrical circuits connected to CPLs. Due to the method is based on linear matrix inequalities (LMI) conditions its numerical solution can be computed efficiently by means of convex programming.

The paper is organized as follows. Section 2 shows some previously published results on NI systems that will be used subsequently. Sections 3 and 4 describe and formalize the problem and present the sufficient condition for the stability, respectively, Section 5 shows two examples: a one-port dc RLC circuit connected to a CPL and a two-port dc RLC with two CPLs. Finally, Section 6 summarizes the contributions and propose some future extensions.

2. PREVIOUS RESULTS

Some previous results that characterize NI systems in the frequency domain and in the state space are reproduced in this section to facilitate the understanding of the subsequent developments.

First, the definition of an NI transfer function matrix and an NI LTI system.

Definition 1. (Petersen and Lanzon (2010)). The square transfer function matrix $R(s)$ is *negative imaginary* if the following conditions are satisfied:

1) All the poles of $R(s)$ lie in the OLHP.
2) for all $\omega \geq 0$,

$$j[R(j\omega) - R^*(j\omega)] \geq 0.$$

A linear time-invariant system is NI if its transfer function matrix is NI.

The NI property can be checked, from a state-space description of the system, using the Lemma 2. Besides, Corollary 3 allows checking, incrementally, if the system is strictly negative imaginary (SNI). It is worth to remark that the main condition to check in Lemma 2 is described by an LMI and, then, it is a convex problem.

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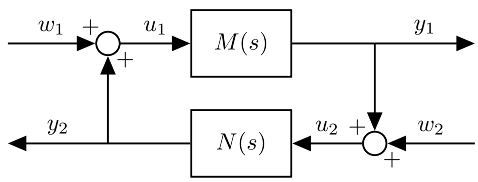

Fig. 1. A positive feedback interconnection.

Lemma 2. (Petersen and Lanzon (2010)).
Consider the minimal state-space system

$$\dot{x} = Ax + Bu, \tag{1}$$
$$y = Cx + Du, \tag{2}$$

Where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, and $D \in \mathbb{R}^{m\times m}$. The system (1), (2) is NI if and only if

1) A has no eigenvalues on the imaginary axis,
2) $D = D^\top$, and
3) there exists a matrix $Y = Y^\top > 0$, $Y \in \mathbb{R}^{n\times n}$, such that

$$AY + YA^\top \leq 0, \quad \textit{and} \quad B + AYC^\top = 0.$$

Corollary 3. (Petersen and Lanzon (2010)). If the minimal state-space system (1), (2), satisfying conditions (1), (2), and (3) from Lemma 2, additionally meets that

4) The transfer function matrix $M(s) = C(sI-A)^{-1}B+D$ is such that $M(s) - M(-s)^\top$ has no transmission zeros on the imaginary axis except possibly at $s = 0$,

then the system is *strictly negative imaginary* (SNI).

Finally, next theorem guarantees the stability of the positive feedback interconnection of Fig. 1.

Theorem 4. (Song et al. (2011)). Given that $M(s)$ is NI and $N(s)$ is SNI, and suppose that $M(\infty)N(\infty) = 0$ and $N(\infty) \geq 0$. Then, the positive-feedback interconnection of these two systems illustrated in Fig. 1 is internally stable if and only if the maximun eigenvalue of the matrix $M(0)N(0)$, denoted by $\bar{\lambda}(M(0)N(0))$, satisfies

$$\bar{\lambda}(M(0)N(0)) < 1. \tag{3}$$

3. PROBLEM STATEMENT

The system of Fig. 2 consists of a linear dc circuit Σ, including dc voltage and current sources, with m CPLs connected to it. These loads do not behave like conventional impedances, instead, their characteristic curves, in the voltage-current plane, are first and third quadrant hyperbolas. Thus, their incremental impedance is negative. The goal is to find an easy to check sufficient condition to determine if the whole system is locally stable for all the possible values of power in the CPLs for which an equilibrium exists in the system. In order to do that, in this Section, the system is defined and, after that, its linearization is calculated.

In the considered networks, see Fig. 2, Σ is a dc LTI RLC network consisting of an arbitrary interconnection of resistors (R), inductors (L), capacitors (C), current sources (I_s), voltage sources (E_s) and LTI magnetic couplings[1]. Assume that Σ has a well-defined minimal order state-space realization, formulated in terms of the d-inductor currents and q-capacitor voltages, as

$$\begin{aligned} \dot{x} &= Ax + Bi + f, \\ v &= Cx + Di, \\ v_{p_i} i_{p_i} &= P_i > 0, \qquad i = 1, ..., m. \end{aligned} \tag{4}$$

where $i = [i_{p_1}, i_{p_2}, ..., i_{p_m}]$ is the input vector, $v = [v_{p_1}, v_{p_2}, ..., v_{p_m}]$ is the output vector, f is a vector of dc voltage and/or current constant sources, P_i are the constant powers absorbed for each port CPL, and $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$ and $D \in \mathbb{R}^{m\times m}$ are constant matrices with $n = d + q$.

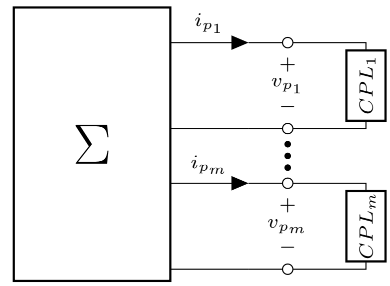

Fig. 2. LTI RLC network including sources connected to m CPLs (m-port case).

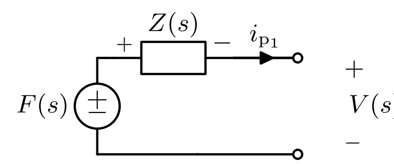

Fig. 3. Thevenin equivalent of the circuit Σ in Fig. 2.

The port input-output representation of Σ in the Laplace domain is

$$V(s) = G(s)\,I(s) + F(s), \tag{5}$$

where s is the Laplace variable, $V(s) = \mathcal{L}\{v(t)\}, I(s) = \mathcal{L}\{i(t)\}$. $G(s) = C(sI - A)^{-1}B + D$ and $F(s) = C(sI - A)^{-1}f$ are rational matrices with real coefficients.

The Thevenin equivalent circuit of Σ, see Fig. 3, is

$$V(s) = -Z(s)\,I(s) + F(s). \tag{6}$$

Matching (5) with (6) we clearly identify $G(s)$ as the equivalent multiport impedance with a negative sign $G(s) = -Z(s)$, and the equivalent multiport voltage source as $F(s)$.

Assuming a given set of fixed CPL powers $P_e := \mathrm{col}\{P_{e_1}, P_{e_2}, ..., P_{e_m}\}$, if the equilibrium of system (4) exists, a set of port voltages $v_e := \mathrm{col}\{v_{e_1}, v_{e_2}, ..., v_{e_m}\}$ and a set of port currents $i_e := \mathrm{col}\{i_{e_1}, i_{e_2}, ..., i_{e_m}\}$ are obtained. Then, the third equation in (4) can be linearized around this equilibrum resulting

$$\Delta i = -K\Delta v + \gamma \Delta P. \tag{7}$$

where $\Delta i := i - i_e$, $\Delta v := v - v_e$ and $\Delta P := P - P_e$ are the incremental vectors of port variables, $K := \mathrm{diag}\{\frac{P_{e_1}}{v_{e_1}^2}, \frac{P_{e_2}}{v_{e_2}^2}, ..., \frac{P_{e_m}}{v_{e_m}^2}\}$ is a constant diagonal gain matrix, and $\gamma := \mathrm{diag}\{\frac{1}{v_{e_1}}, \frac{1}{v_{e_2}}, ..., \frac{1}{v_{e_m}}\}$ is the gain of incremental power disturbances. Obviously, in equation (7) $-K$ represents the negative incremental admitance of the CPLs. Applying Laplace transform to (7), we obtain the linear

[1] For the sake of brevity, the time arguments, $t \in \mathbb{R}$, of the circuit variables are not included unless necessary.