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Virtual Bidding: Equilibrium, Learning, and the Wisdom of Crowds [★]

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Abstract: We present a theoretical analysis of virtual bidding in a stylized model of a single bus, two-settlement electricity market. North-American ISOs typically take a conservative approach to uncertainty, scheduling supply myopically in day-ahead (DA) markets to meet expected demand, neglecting the subsequent cost of recourse required to correct imbalances in the real-time (RT) market. This can result in generation costs that far exceed the minimum expected cost of supply. We explore the idea that virtual bidding can mitigate this excess cost incurred by myopic scheduling on the part of the ISO. Adopting a game-theoretic model of virtual bidding, we show that as the number of virtual bidders increases, the equilibrium market outcome tends to the socially optimal DA schedule, and prices converge between the DA and RT markets. We additionally analyze the effects of virtual bidding on social welfare and the variance of the price spread. Finally, we establish a repeated game formulation of virtual bidding, and investigate simple learning strategies for virtual bidders that guarantee convergence to the Nash equilibrium.

Keywords: Electric Power Systems, Game Theory, Economics, Stochastic Programming, Learning Algorithms, Asymptotic Analysis

1. INTRODUCTION

In electricity markets, virtual bidding (VB) allows market participants to buy and sell electricity without the obligation to physically produce or consume it. This opens up market participation to financial entities or third parties without generation or load assets, allowing them to take advantage of arbitrage opportunities and promote market liquidity. VB is similar in nature to futures trading in more traditional commodity markets, where contracts are settled financially and no physical delivery takes place.

Deregulated electricity markets are typically characterized by centralized multi-settlement markets administered by an independent system operator (ISO). More specifically these markets have both a day-ahead (DA) and real-time (RT) market. In the DA market, the ISO collects demand bids and supply offers from participants and, based on the expected transmission network conditions, determines an economic unit commitment and dispatch with associated locational marginal prices (LMPs) for each hour of the next day. A similar economic dispatch procedure is conducted in the RT market, but in response to real-time system conditions, typically at five to fifteen minute intervals. The important distinction between the two markets is that cleared DA schedules are just financial contracts that can be settled at real-time prices, whereas the RT market represents physical delivery of energy *i.e.* no power flows in the DA market. It is this fact that enables the inclusion of VB that is not backed by physical assets in electricity markets. [1] A more complete discussion of these issues can be found in Hogan (2016).

A virtual bid in such a market structure is comprised of a buy (sell) bid in the DA market, matched by a sell (buy) offer in the RT market, such that any position taken up in the DA is completely liquidated in the RT market, with no obligation to physically produce or consume electricity. This allows virtual bidders to arbitrage the price difference between the DA and RT markets. This should in general cause the DA and RT prices to converge in expectation, as any price gap can be exploited by a risk neutral speculator. This is why VB is sometimes referred to as convergence bidding. It is also important here to highlight the difference between explicit and implicit VB. In the absence of an explicit VB mechanism, participants backed by physical assets can still make implicit virtual bids, for example bidding more capacity than they have available into the DA market and then purchasing the shortfall on the spot market in real time. Implicit VB can cause market power issues, and compromise the integrity of load and generation forecasts. Allowing a mechanism for explicit VB, as described above, goes some way to mitigating these issues. More broadly, whenever we discuss VB in this paper, we are referring to explicit VB.

The benefits of virtual bidding are discussed at length in Hogan (2016); Celebi et al. (2010); Isemonger (2006), and are generally characterized as: improved liquidity, mitigation of market power, improved market efficiency and price formation, reduced price volatility, and providing market participants with the ability to hedge price risk. A poten-

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[1] VB is implemented in the majority of North-American ISOs, including PJM, NYISO, ISO-NE, MISO, and CAISO.

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tial downside of virtual bidding highlighted in the above works is the incentive for a virtual bidder in possession of a bilateral or external position to influence the profitability of this position through virtual trades. This is of particular relevance to those traders in possession of financial transmission rights (FTRs), as described in Ledgerwood and Pfeifenberger (2013), to the effect that both ISO-NE and PJM enforce revenue capping when a participant makes a virtual bid which affects its own FTR revenue stream. Parsons et al. (2015) also suggest that virtual traders can exploit approximations in market designs to make profits without improving system operation, for example real-time ramping requirements that are not considered in the DA market. Some attempts have been made to quantify the efficiency effects of virtual bidding through empirical studies testing for the existence of profitable bidding strategies. See Saravia (2003); Borenstein et al. (2008); Li et al. (2015); Jha and Wolak (2015).

In this paper we focus on the ability of virtual bidding to improve outcomes in electricity markets with uncertainty. Hogan (2016) emphasizes this as one of the most valuable aspects of VB, yet also highlights the lack of rigorous work or analysis in this area, mainly due to the complexity involved. Modern electricity markets face increasing uncertainty in both supply and demand with a growing penetration of renewable and distributed generation. ISOs typically take a conservative approach to uncertainty, scheduling supply myopically in the DA market to meet expected demand, and neglecting the subsequent cost of recourse required to correct imbalances in the real-time (RT) market. They also hold significant reserve margins to manage large deviations or deal with contingencies. This deterministic approach to power markets provides reliable and secure system operation, but it can be costly. Recent advances in stochastic and robust optimization have shown that significant cost reductions can be achieved by more explicitly incorporating uncertainty into market clearing algorithms. See Bertsimas et al. (2013); Munoz-Alvarez et al. (2014); Hreinsson et al. (2015). Such approaches are tractable for real, large-scale, power systems; however, they face resistance from ISOs and system operators due to their perceived complexity, opacity, and reduction in system reliability.

We propose the novel thesis that, under certain assumptions, deterministic system operation with virtual bidding approximates the results of stochastic system operation, obviating the need for implementing new market algorithms. We demonstrate this result on a stylized model of a single bus, two-settlement electricity market. While a simple model, the results are instructive and point the way to models that more closely approximate the true operation of real power systems in future work. Our model is similar in nature to that proposed by Tang et al. (2016), although the equilibrium analysis, welfare analysis, and learning dynamics presented here are novel. All of these analyses are shown to depend on the accuracy of the aggregate beliefs of the population of virtual bidders. In short, the wisdom of the crowd. Our contributions are as follows:

- We characterize the unique, pure strategy Nash equilibrium of a population of profit-maximizing virtual bidders with heterogeneous beliefs about the market in which they participate.
- We show that as the number of virtual bidders increases, the DA ISO schedule approaches the socially optimal schedule, and prices converge in expectation between the DA and RT markets.
- We investigate simple learning strategies for individual speculators and characterize conditions under which they converge to the unique Nash equilibrium.

Organization: The remainder of the paper is organized as follows. In Section 2, we formulate a model of the two-settlement market and the virtual bidding mechanism. In Section 3 we characterize the pure Nash equilibrium among virtual bidders, and discuss its effect on social welfare. In Section 4 we propose simple learning dynamics under which virtual bidders reach the Nash equilibrium, and Section 5 concludes.

Notation: Denote by $\mathbb{R}$ and $\mathbb{R}_+$ the sets of real and nonnegative real numbers, respectively. Denote the transpose of a vector $x \in \mathbb{R}^n$ by $x^\top$. Let $x_{-i} = (x_1, .., x_{i-1}, x_{i+1}, .., x_n) \in \mathbb{R}^{n-1}$ be the vector including all but the i^{th} element of x. Denote by $\mathbf{1}$ the vector of all ones, and by $\mathbf{E} := \mathbf{1}\mathbf{1}^\top$ a square matrix of all ones. Denote by $\operatorname{diag}(x_1, \ldots, x_n)$ the diagonal matrix with diagonal elements $\{x_i\}_{i=1}^n$.

2. MARKET MODEL

We consider a simplified model of a two-settlement electricity market administered by an independent system operator (ISO) for a copper plate power system.[2] The electricity market is cleared in two stages: day-ahead (DA) and real-time (RT). In the DA market, the ISO must determine an initial dispatch of supply subject to uncertainty in the eventual realization of demand, which we assume to be perfectly inelastic and denote by $D \in \mathbb{R}_+$. We describe uncertainty in the ISO's prior belief about demand by modeling D as a random variable with mean $\mu := \mathbb{E}[D]$ and variance $\sigma^2 := \operatorname{Var}(D)$.

The ability to schedule supply in the DA market is essential, as certain generation resources (e.g., coal and nuclear) have limited ramping capability, and must therefore be scheduled well in advance of the required delivery time. We define the production cost in the DA market according to a convex quadratic function of the form

$$C_{\text{DA}}(x) := \frac{1}{2}\alpha x^2,$$

for all production levels $x \geq 0$. Here, $\alpha > 0$ is assumed to be fixed and known by the ISO.

In RT market, demand is realized, and any mismatch between supply scheduled in the DA market, say x, and the realized demand D must be compensated through an adjustment of supply in the amount of $D - x$. The subsequent balancing cost incurred in the RT market is assumed to be a convex quadratic function of the form

$$C_{\text{RT}}(D - x) := \frac{1}{2}\beta(D - x)^2 + \gamma(D - x),$$

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[2] We use the term *copper plate* here to imply a lossless, unconstrained transmission system.