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Robust H_∞ Control for Autonomous Scooters

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Abstract: This paper studies the trajectory tracking problem for the nonlinear model of a scooter and presents a robust H_∞ controller based on measurements of the tracking errors, the roll angle, the yaw angle and the steering angle. The study first introduces the full nonlinear model developed in Autosim which has 12 degrees of freedom. This is far more complex than a simple bicycle model and provides a good description of the scooter. Then a robust H_∞ controller based on the linearization of the nonlinear model is designed. Finally, the effectiveness of the controller is verified by means of two case studies.

Keywords: Motorcycle, Nonlinear Dynamics, Auto-driving, Trajectory Tracking, Robust Control

1. INTRODUCTION

With the development of technology, autonomous cars and "smart cars" have attracted significant researchers' attention. "Google cars" are one of the most successful applications of this research. On the contrary, the research on autonomous two-wheeled vehicles, such as motorcycles and scooters, is not as active because the system is more complex. In particular, one major difficulty in the control of two-wheeled vehicles is related to the balance issue.

Even though cars are widely used in daily life, scooters are popular in some areas, especially developing countries where roads are not well-built. Furthermore, scooters are much more useful than four-wheeled vehicles in narrow tracks due to their small size. Even in large cities, people may prefer scooters during rush hours. Moreover, the light weight property of scooters provides other attractive features, such as high energy efficiency.

Four typical models are commonly used in the literature to study the behaviour of two-wheeled vehicles. The first one is the simplified bicycle model in Astrom et al. (2005); Limebeer and Sharp (2006); Lowell and McKell (1982), the dynamics of which is second order. The model captures the main relationship between the lean angle and the steering angle: this makes the control design straightforward. However, the model is too simple to be used for applications. Another popular model used in the control of two-wheeled vehicles is the benchmark model described in Schwab et al. (2005); Meijaard et al. (2007). It consist of four rigid symmetric ideally-hinged parts: the main body, the front assembly and the two wheels. The model has a seven-dimensional configuration space and three degrees of freedom. Even though the linear model can be used to calculate the speed at which the bicycle lean and steering are self-stable, it only models accurately small perturbations from the straight running case. A third type of two-wheeled control-oriented models which combine the scooter model with motor and sensor models have also been considered in the literature, see Brankovic et al. (2015). Finally, a well-known model for two-wheeled vehicles is the multi-body model developed by Sharp and coworkers, see Sharp and Limebeer (2001); Sharp et al. (2004); Evangelou et al. (2012) through AutoSim, a platform used to build the symbolic model of a multi-body system. In this model the complex geometric description of the steering system and the tyre forces are included.

Researchers and engineers have studied the fore-mentioned models to design controllers for autonomous bicycles, motorcycles and scooters. The paper Yi et al. (2006) has presented a nonlinear control algorithm to calculate the rear-wheel driving force and the steering angular velocity while guaranteeing the balance of the motorcycle, while Frezza et al. (2004) have proposed a strategy for the steering angle and the longitudinal forces based on Model Predictive Control. Another control scheme to track the roll angle and the forward velocity based on a second-order sliding mode control has been introduced in Defoort and Murakami (2009). An active steering compensator is designed in Evangelou et al. (2006) to replace the steering damper and to improve the dynamic behaviour of the vehicle. Furthermore, the path tracking problem has been translated into a roll angle tracking problem by preview in Dao and Chen (2011), where roll angle tracking has been achieved by state feedback control and integral action. Even though various control methods have been used to

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design the controller, none of them is designed for the complex multi-body model to track an (x, y) trajectory. In addition, in existing works the longitudinal control (that is the control of the braking forces, the driving torque and the forward speed) and the lateral control (that is the control of the lean angle, the lean angular velocity, the steering angle and the steering torque) are usually independent. In the paper a coupled longitudinal and lateral controller to track any (feasible) (x, y) reference trajectory is proposed.

The rest of the paper is organized as follows. Sections 2 and 3 describe the model studied and formulate the trajectory tracking problem, respectively. Several assumptions, which hold throughout the paper, are also given in Section 3. The design of the robust output-feedback controller for the scooter based on H_∞ synthesis is presented in Section 4. In Section 5 two case studies are given to illustrate how the controller works. Finally, conclusions and suggestions for future work are discussed in Section 6.

2. SCOOTER MODELING

The model we consider is based on the structure of the BMW-C1 scooter. Many of the motorcycle studies are based on the simple bicycle model Astrom et al. (2005), and therefore neglect the effect of multi-body connections. The model studied in this paper is a multi-body system which contains seven major components: the main body, the rear suspension, the front frame twist body, the steering frame, the front suspension and the two wheels. A rider is considered to be sitting on the scooter but without interfering with the control. As a result, the rider mass and inertia are lumped with the main body of the scooter: that is reasonable since the rider's movements relative to the vehicle on a BMW-C1 are fairly restricted because of the seat belts.

The scooter model is adapted from the Suzuki GSX-R1000K1 model in Sharp et al. (2004), with the monoshock rear suspension of the Suzuki replaced by a twin shock rear suspension, as founded on the BMW-C1 scooter. Furthermore, steering friction is added to the scooter model to represent the friction in the steering actuator system that is used to drive the vehicle autonomously.

2.1 State Variables, Inputs and Measurements

The full nonlinear model of the scooter is described by 26 nonlinear ordinary differential equations, representing its kinematic and dynamic behaviour. It has 12 degrees of freedom and 14 generalized coordinates. The 12 multi-body coordinates are the main body center position (x_c, y_c, z_c), the main body yaw angle ψ, pitch angle μ and roll angle ϕ, the twist angle of the frame twist body, the steering angle of the steering frame, the deflection of the front suspension body, the rotation of the swinging arm and the rotation of the two wheels. The two auxiliary coordinates are the rear tyre side-slip r_β and the front tyre side-slip f_β, by which first order relaxation type tyre forces are introduced. In addition, 12 independent speed variables, each for each degree of freedom, are used to describe the speed of each body. They are the main body velocities projected in the longitudinal, the lateral and the vertical directions, x_v, y_v and z_v, the main body yaw rate ψ_v, pitch rate μ_v and roll rate ϕ_v and the rotation or the translation speed of each child body relative to its parent body.

The motorcycle has four control inputs: the driving torque, the steering torque and the front and the rear braking forces. In the paper we focus on the first two inputs since brakes are used only in cases when we want to slow down the scooter sharply, which is not common in trajectory tracking cases. In addition, in our model driving torques have negative values and braking forces can be translated to positive driving torques as illustrated in Limebeer et al. (2001).

In the paper we study the control of the scooter based on five measurements: the (x, y) position of the front wheel contact point, the yaw angle ψ, the roll angle ϕ and the steering angle δ. We have chosen to measure the (x, y) position of the front wheel contact point rather than the position of the mass center of the main body because the reference trajectory we aim to track is defined on the ground. The three angular feedback signals are three essential states of the system which need to be monitored during auto-driving tests.

To sum up, the dynamics of the full nonlinear state space model of the scooter can be written as

$$\begin{aligned} \dot{s} &= f(s, u), \\ m &= g(s, u), \end{aligned} \tag{1}$$

where $s(t) \in \mathbb{R}^{26}$ denotes the state vector, $u(t) = [T_{str}(t), T_{drv}(t)]^T \in \mathbb{R}^2$ and $m = [x(t), y(t), \phi(t), \delta(t), \psi(t)]^T \in \mathbb{R}^5$ represent the control inputs and the measurements, respectively. Note that T_{str} and T_{drv} denote the steering torque and the rear wheel driving torque, respectively. For simulation purposes, in Section 5, Gaussian measurement noise has been added to the model as follows:

$$\begin{aligned} \dot{s} &= f(s, u), \\ m &= g(s, u) + n, \end{aligned}$$

where $n = [n_x, n_y, n_\phi, n_\delta, n_\psi]^T$.

2.2 Model Validation

Stability analysis for the open-loop system is a simple and commonly used way to validate the model. In this subsection the stability analysis of the scooter in straight running condition is presented in terms of a root-locus diagram, see Fig. 1, showing the eigenvalues of the "uncontrolled" linearized system in straight running conditions for a range of vehicle forward velocities: from $v = 3\ m/s$ to 30 m/s.

From Fig. 1 it is easy to identify the well-known modes of two-wheel vehicles: the wobble mode and the weave mode. The wobble mode has lower frequency at higher speed. Moreover, the wobble instability "decreases" as the forward velocity "increases" from 3 m/s to 12 m/s, while it increases as v increases from 12 m/s to 30 m/s. It hits the bound of the stability region when the forward velocity reaches its maximum value (30 m/s). The figure also shows that the weave mode has very low frequency at low speeds (*i.e.* less than $0.5Hz$) and the frequency increases to about 3 Hz as the speed increases. In addition, the weave mode is unstable at low speed and becomes stable at medium