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# Path Following Control Tuning for an Autonomous Unmanned Quadrotor Using Particle Swarm Optimization

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**Abstract**: The goal of this work is to present the particle swarm optimization application for quadrotor attitude and path following control tuning. To perform this task a path following feed-forward plus proportional-derivative control strategy was implemented, using particle swarm for tuning gains, and root mean square error for validating. The basic of quadrotor kinematics and dynamics model will be presented. Path planning will be executed through Euler-Lagrange equations to minimize the snap cost function and guarantee a soft trajectory through a set of intermediary waypoints. The reliability of this approach will be tested through several simulations.



*Keywords:* UAVs, autonomous robotic, trajectory and & planning, trajectory tracking & path following.

## 1. INTRODUCTION

Unmanned aerial vehicles (UAVs) have proved to be a promising solution in recent years for a wide range of applications such as reconnaissance, mapping, firefighting, disaster relief, search and tracking operations (Tisdale et al., 2009, Fu et al., 2012). Its popularity has stemmed from its simple construction compared with conventional helicopters (Dierks and Jagannathan, 2010).

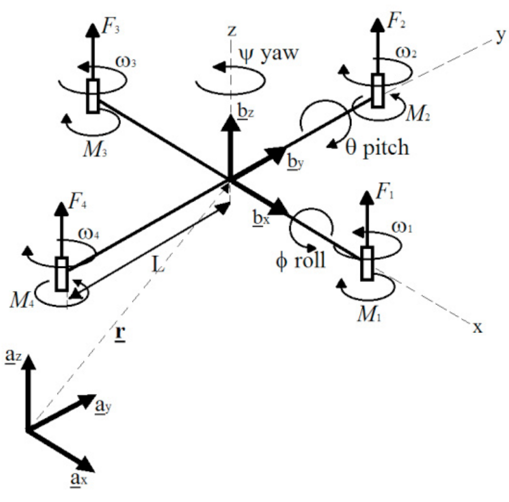


Fig. 1. *Quadrotor configuration, body fixed frame B and earth inertial frame A*

A quadrotor employs fixed pitch rotors, whose propeller speeds can be adjusted to achieve control (Dierks and Jagannathan, 2010). Each pair of propellers turn in opposite directions (Fig. 1). Setting different rotor speeds in each pair the quadrotor produces roll or pitch rotation with corresponding lateral motion. Yaw rotation results from torque difference between propellers (Bouabdallah and Siegwart, 2005). The current trend is to design small flying vehicles capable of hovering with less oscillation (Chiou et al., 2016).

Quadrotor is an unstable nonlinear system, therefore the development of a high performance controller is important (Basri et al., 2015). Different control methods have been researched, including PID hovering control (Luukkonen, 2011), sliding mode techniques for adjusting PD controllers (Dikmen et al., 2009), back-stepping and sliding-mode control (Bouabdallah and Siegwart, 2005, Zuo, 2010) and nonlinear controllers. Research areas include trajectory planning (Zuo, 2010) and heuristic methods for path planning (Luukkonen, 2011) for one or a group of quadrotors. Real-time trajectory planners have been researched in the last years using Genetic Algorithm and Particle Swarm Optimization (PSO) (Roberge et al., 2013), and new variations of PSO (Fu et al., 2012).

Quadrotor has six degrees of freedom (DOF) and requires robust control schemes to alleviate model mismatches, wind disturbances, measurement noise ,etc.

PSO is a technique based on stochastic and population-based adaptive optimization and swarm intelligence. It has been used to optimize fuzzy PID controller gains and solve quadrotor attitude control systems (Chiou et al., 2016).

This paper presents the basic quadrotor model, path planning, attitude control and path following control tuned with a PSO algorithm. Parameter values from (Tayebi and McGilvray, 2004) are used in the simulations.


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## 2. QUADROTOR MODEL

Quadrotors are highly dynamic systems whose model includes gyroscopic effects from rigid body rotation in space (Bouabdallah and Siegwart, 2005). Newton-Euler equations are used to describe a quadrotor's dynamics and kinematics (Beard, 2008). In this dynamic representation, only the gravitational force and the thrust contribute to produce quadrotor's linear and angular accelerations.

Let us consider earth fixed frame $A$ and body fixed frame $B$ (Fig. 1). Using Euler angles *roll* $\phi$, *pitch* $\theta$ and *yaw* $\psi$ for parametrization, the quadrotor orientation in space is given by a rotation from $B$ to $A$, using the rotation matrix $\underline{\boldsymbol{R}}$ (1).

$$\underline{\boldsymbol{R}} = \begin{bmatrix} c\psi\, c\theta - s\phi\, s\psi\, s\theta & -c\phi\, s\psi & c\psi\, s\theta + c\theta\, s\phi\, s\psi \\ c\theta\, s\psi + c\psi\, s\phi\, s\theta & c\phi\, c\psi & s\psi\, s\theta - c\theta\, s\phi\, c\psi \\ -c\phi\, s\theta & s\phi & c\phi\, c\theta \end{bmatrix} \quad (1)$$

$$\underline{\boldsymbol{\eta}} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix}; \quad \underline{\boldsymbol{r}} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \quad \underline{\boldsymbol{\nu}} = \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (1)$$

Where $\underline{\boldsymbol{r}}$ is the position vector (Fig. 1) and $\underline{\boldsymbol{\nu}}$ is the angular velocities ($p$, $q$ and $r$) vector (2).

Consider $\underline{\boldsymbol{I}}$ as the inertial matrix (3), $L$ the quadrotor's arm length (Fig. 1), $m$ its mass, and $g$ the earth's gravity. This model doesn't consider the air resistance drag force (Luukkonen, 2011):

$$\underline{\boldsymbol{I}} = \begin{bmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{bmatrix}; \quad (3)$$

$$u_1 = F_1 + F_2 + F_3 + F_4 \quad (4)$$

$$\underline{\ddot{\boldsymbol{r}}} = \begin{bmatrix} \ddot{x} \\ \ddot{y} \\ \ddot{z} \end{bmatrix} = \frac{1}{m}\left\{ \begin{bmatrix} 0 \\ 0 \\ -mg \end{bmatrix} + \underline{\boldsymbol{R}} \begin{bmatrix} 0 \\ 0 \\ u_1 \end{bmatrix} \right\}; \quad (5)$$

$$\underline{\boldsymbol{u_2}} = \begin{bmatrix} L(F_2 - F_4) \\ L(F_3 - F_1) \\ M_1 - M_2 + M_3 - M_4 \end{bmatrix} \quad (6)$$

$$\underline{\dot{\boldsymbol{\nu}}} = \begin{bmatrix} \dot{p} \\ \dot{q} \\ \dot{r} \end{bmatrix} = \underline{\boldsymbol{I}}^{-1} \left\{ \underline{\boldsymbol{u_2}} - \begin{bmatrix} p \\ q \\ r \end{bmatrix} \times \underline{\boldsymbol{I}} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \right\}; \quad (7)$$

$$\underline{\dot{\boldsymbol{\eta}}} = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \underline{\boldsymbol{T}}^{-1} \cdot \underline{\boldsymbol{\nu}} = \begin{bmatrix} c\theta & 0 & -c\phi\, s\theta \\ 0 & 1 & s\phi \\ s\theta & 0 & c\phi\, c\theta \end{bmatrix}^{-1} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \quad (8)$$

Commanding the quadrotor's motors (and consequently the forces $F_i$) through signals $u_1$ and $\underline{\boldsymbol{u_2}}$ it is possible to develop path following and attitude control. Position vector $\underline{\boldsymbol{r}}$ and angular acceleration vector $\underline{\dot{\boldsymbol{\nu}}}$ are adjusted in (5) and (7).

## 3. PATH PLANNING FOR A SINGLE QUADROTOR

Path planning for UAV's is one of the most important parts of mission planning. It must constantly update its trajectory because of terrain conditions (Roberge et al., 2013). It consists of generating a path between an initial and a desired destination to obtain an optimal performance under specific constraints like minimizing the distance, altitude, fuel consumption, radar exposure etc. (Zheng et al., 2005). Researchers have published works with deterministic and nondeterministic algorithms (Roberge et al., 2013). Evolutionary algorithms are more powerful in terms of cost, stability and convergence speed when flying above some static threat environments (Fu et al., 2012).

For UAV path planning the robot must plan its own optimized trajectory according some optimizing criteria (9).

$$x_{des}(t) = \underset{x(t)}{arg\min} \int_0^T \mathcal{L}\left(t, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)}\right) dt \quad (9)$$

Since it is a fourth order dynamic system, we plan its soft trajectory along a set of waypoints minimizing the snap (fourth derivative of position). For a trajectory with initial and final fixed points the Euler-Lagrange function that minimizes the snap is (10).

$$\mathcal{L}\left(t, \dot{x}, \ddot{x}, \dddot{x}, x^{(4)}\right) = \left(\frac{d^4x}{dt^4}\right)^2 \quad (10)$$

$$\frac{\partial\mathcal{L}}{\partial x} - \frac{d}{dt}\left(\frac{\partial\mathcal{L}}{\partial\dot{x}}\right) + \frac{d^2}{dt^2}\left(\frac{\partial\mathcal{L}}{\partial\ddot{x}}\right) - \frac{d^3}{dt^3}\left(\frac{\partial\mathcal{L}}{\partial\dddot{x}}\right) + \frac{d^4}{dt^4}\left(\frac{\partial\mathcal{L}}{\partial x^{(4)}}\right) = 0 \quad (11)$$

As Euler-Lagrange equation in (10) has just fourth derivative elements, the first partial derivatives in (11) are zero.

$$\frac{d^8x}{dt^8} = 0 \quad (12)$$

Solving equation (12) we obtain the trajectory equation that minimizes snap, a seventh order polynomial (13).

$$\begin{aligned} x_{des}(t) = c_{1,7}t^7 + c_{1,6}t^6 + c_{1,5}t^5 + c_{1,4}t^4 + \cdots \\ \cdots + c_{1,3}t^3 + c_{1,2}t^2 + c_{1,1}t^1 + c_{1,0} \end{aligned} \quad (13)$$

To find the eight constants in (13) eight restrictions must be satisfied: position, velocity, acceleration and jerk boundary conditions in the initial and the end, as shown in Table 1.

**Table 1. Boundary Conditions**

| Time | Position | Velocity | Acceleration | Jerk |
|---|---|---|---|---|
| 0 | $x_0$ | 0 | 0 | 0 |
| $T$ | $x_T$ | 0 | 0 | 0 |

Time equations for each variable must satisfy the eight restrictions in the beginning and the end of trajectory (14).

$$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ T^7 & T^6 & T^5 & T^4 & T^3 & T^2 & T^1 & T^0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 7T^6 & 6T^5 & 5T^4 & 4T^3 & 3T^2 & 2T^1 & T^0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \\ 42T^5 & 30T^4 & 20T^3 & 12T^2 & 6T^1 & 2T^0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 6 & 0 & 0 & 0 \\ 210T^4 & 120T^3 & 60T^2 & 24T^1 & 6T^0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} c_{1,7} \\ c_{1,6} \\ c_{1,5} \\ c_{1,4} \\ c_{1,3} \\ c_{1,2} \\ c_{1,1} \\ c_{1,0} \end{bmatrix} = \begin{bmatrix} x_0 \\ x_T \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

Following this we are able to calculate the coefficients matrix for the desired trajectory $\underline{\boldsymbol{r}}_{des} = [x_{des} \quad y_{des} \quad z_{des}]^T$ and orientation $\psi_{des}$ equations (15).