Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 50-1 (2017) 349–354

H_∞ - Optimal Control over erasure channel

Abhijit Mazumdar * **Srinivasan Krishnaswamy** *
Somanath Majhi *

* *Dept. of Electronics and Electrical Engineering*
Indian Institute of Technology Guwahati, Guwahati-781039, India.
E-mails:{abhijit.mazumdar, srinikris, smajhi}@iitg.ernet.in.

Abstract: In this paper, the optimal H_∞ control of a linear time invariant (LTI) networked control system (NCS) over a lossy network is analyzed. Specifically, we consider a TCP-like data network where packet drops are modelled using a two-state Markov process. By employing the theory of dynamic games, a state-feedback H_∞ controller is obtained. It is shown that, subjected to a suitable H_∞ disturbance attenuation level and suitable packet loss probabilities, such a controller exists for both finite and infinite horizon cases. We also show the asymptotic stability of the NCS in the mean-square sense. Finally a numerical example is presented to demonstrate our work.

Keywords: H_∞ control, networked control systems, packet loss, TCP, UDP

1. INTRODUCTION

Development in communication and networking technologies has made possible to design spatially distributed systems, where communication between the different subsystems (sensors, actuators and controllers) is achieved through a multipurpose shared network. These types of systems are termed as Networked Control Systems (NCSs). However, the insertion of the network introduces data packet losses (Schenato et al. (2007)).

Packet loss is a significant issue that results from transmission errors in physical network links or from buffer overflows due to congestion. It degrades the performance of an NCS. Packet loss can be modelled in many ways such as Bernoulli process (Schenato et al. (2007)), Markov chain model (Mo et al. (2013)). Most of the works on NCSs consider the case where packet losses are modelled by i.i.d. Bernoulli processes with a few notable exceptions (Mo et al. (2013)), (Huang and Dey (2007)). However, the current packet cases (dropped and received successfully) can affect the future packet cases. The Markovian packet drop model can capture this effect and represent a more realistic packet loss model (Schenato (2009)). Communication networks can be categorized based on whether it acknowlege the packet reception (TCP-like protocols) or not (UDP-like protocols)

Schenato et al. (2007) deals with optimal control in the context of NCSs assuming packet loss to be a Bernoulli process. It is proved that for TCP-like protocols, there exists a threshold probability for successful packet delivery below which the controller cannot stabilize the system. Further, it was proved that the separation principle holds for TCP-like protocols while it does not hold for UDP-like protocols. Garone et al. (2012) extends this result in the TCP-like framework to the case where there are multiple memoryless erasure channels between the sensors, the controller and the actuators.

The design of stationary H_∞ control problem for NCSs has been discussed in (Wang et al. (2007)) & (Sahebsara et al. (2008)). In Seiler and Sengupta (2005) and Gonçalves et al. (2009), time invariant H_∞ controllers are designed using the Markov Jump Linear System (MJLS) theory. The H_∞ control problem for NCSs with control packet loss modelled as a Bernoulli process is investigated in Moon and Başar (2013). Shoukry et al. (2013) consider the minimax control over unreliable network and designed a resilient controller to deal with the time varying delay induced by attack on the scheduling algorithm. Moon and Basar (2014) & Moon and Başar (2015) deal with the minimax control of NCSs with both sensor and control packet losses. Both these papers consider a Bernoulli binary distribution for packet loss.

In this paper, we study the H_∞ optimal control problem for a linear time invariant (LTI) systems with Markovian packet loss. Here we consider the case where the channels have memory which can be modelled using a particular Gilbert-Elliot model which is a two-state Markov chain model (Mo et al. (2013)). In our work, we have followed the zero-input strategy, i.e. if a control packet from the controller to the actuator is lost then the actuator applies no input signal. Existence of the saddle point is proved under some conditions, for both finite horizon and infinite horizon. Finally we prove the asymptotic stability of the system.

The paper is organized as follows. In Section 2, H_∞ optimal control problem with control packet loss is formulated as dynamic game problem. Both finite horizon and infinite horizon H_∞ controllers are designed in Section 3. Different critical parameters are characterized and stability analysis is also provided. In Section 4, we consider a numerical example to demonstrate the results of section 3. Finally we conclude the paper in Section 5.

10.1016/j.ifacol.2017.08.157

2. PROBLEM FORMULATION

Consider the following linear system

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k^a + D_1 w_k, \\ z_k &= Cx_k + Du_k^a, \end{aligned} \tag{1}$$

where $x_k \in \mathcal{X} \subseteq \mathbb{R}^n$ is the state vector, $u_k^a \in \mathcal{U} \subseteq \mathbb{R}^m$ is the control input applied by the actuator, $w_k \in \mathcal{W} \subseteq \mathcal{L}_2([0,\infty), \mathbb{R}^s)$ is disturbance input to be rejected, $z_k \in \mathbb{R}^p$ is the controlled output. The matrices A, B, D_1, C and D are time invariant, and of appropriate dimensions. It is assumed that state information can be measured directly.

We consider the Gilbert-Elliott channel model which has a two-state Markov chain. The state transition diagram of the model is as shown in figure 1. Now, if $u_k \in \mathbb{R}^m$ is the

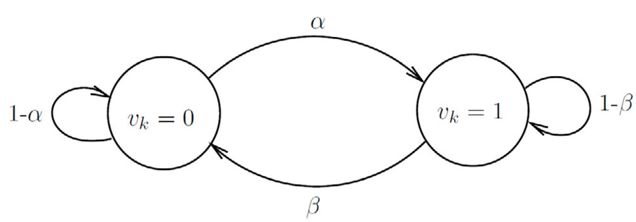

Fig. 1. Gilbert-Elliott channel model (Mo et al. (2013))

control input generated by the controller and being sent to the actuator through the network then,

$$u_k^a = v_k u_k \tag{2}$$

where, the random variable v_k represents the packet loss condition. If a packet send by the controller is lost then $v_k = 0$ and if it is recieved successfully by the actuator then $v_k = 1$. The transition probability matrix is as follows:

$$\begin{bmatrix} P(v_{k+1}=0|v_k=0) & P(v_{k+1}=1|v_k=0) \\ P(v_{k+1}=0|v_k=1) & P(v_{k+1}=1|v_k=1) \end{bmatrix} = \begin{bmatrix} 1-\alpha & \alpha \\ \beta & 1-\beta \end{bmatrix} \tag{3}$$

Note 1: At a particular time index k, if there is no previous information regarding the packet loss status (v_{k-1}), then the probabality of having a packet loss i.e. $v_k = 0$ will always be same and is given by: $P(v_k = 0) = \beta/(\alpha+\beta)$, $\forall k$ (Mo et al. (2013)).

Under TCP-like protocol, we can define the information set $\mathcal{I}_k$ available to the controller at each k as:

$$\mathcal{I}_k = \{x_0, x_1, ..., x_k, v_0, v_1, ..., v_{k-1}\} \tag{4}$$

In the above expression, the control packet acknowledgement sequence $\{v_0, v_1, ..., v_{k-1}\}$ containing in the information set $\mathcal{I}_k$ makes it distict from the information set under UDP-like protocol. $\mathcal{U}$ and $\mathcal{W}$ are the sets of admissible control and disturbance policies, respectively. We can define sequences of control policy $\mathbf{u}^k$ and disturbance policy $\mathbf{w}^k$ as follows: $\mathbf{u}^k = \{u_0, u_1, ..., u_k\}$, $\mathbf{w}^k = \{w_0, w_1, ..., w_k\}$.

Definition 1: The system (1) is said to attain H_∞ disturbance attenuation level γ if

$$\sum_{k=0}^{N} \mathbb{E}\left\{||z_k||^2\right\} \leq \gamma^2 \sum_{k=0}^{N} ||w_k||^2$$

where $||.||$ is the Euclidean norm with the notation $||z_k|| = (z_k^T z_k)^{1/2}$. The attenuation level γ represents the effect of disturbance w_k on the controlled output z_k. In the following section we will design an optimal controller subjected to cost function (5), which maintains the prescribed H_∞ disturbance attenuation level γ for system (1) with different control packet loss probabilities.

Now let us consider the following cost function:

$$J_N(\mathbf{u}^{N-1}, \mathbf{w}^{N-1}) = \mathbb{E}\Big[||x_N||^2_{W_N} + \sum_{k=0}^{N-1} ||z_k||^2 - \gamma^2 ||w_k||^2 |\mathcal{I}_N\Big] \tag{5}$$

By using equation (1) we can rewrite the above cost function as:

$$J_N(\mathbf{u}^{N-1}, \mathbf{w}^{N-1}) = \mathbb{E}\Big[||x_N||^2_{W_N} + \sum_{k=0}^{N-1} ||x_k||^2_W + v_k ||u_k||^2_R - \gamma^2 ||w_k||^2 |\mathcal{I}_N\Big] \tag{6}$$

here $||x_k||^2_{W_N} := x_k^T W_N x_k$, $W = C^T C \geq 0$, $R = D^T D > 0$, γ is the disturbance attenuation level. It is assumed that $C^T D = 0$ i.e. there are no cross product terms in the cost function. On the other hand $R = D^T D > 0$ implies that the optimal control problem is nonsingular.

In the above two-player zero-sum game (6), control policy $\mathbf{u}^k$ is the minimizing player and disturbance policy $\mathbf{w}^k$ is the maximizing player. The above game admits a solution if there exist a saddle point $\big(\mathbf{u}^{*k}, \mathbf{w}^{*k}\big)$ which satisfies:

$$J_k(\mathbf{u}^{*k}, \mathbf{w}^k) \leq J_k(\mathbf{u}^{*k}, \mathbf{w}^{*k}) \leq J_k(\mathbf{u}^k, \mathbf{w}^{*k}) \tag{7}$$

Our main goal in this paper is to find a saddle point $(\mathbf{u}^{*k}, \mathbf{w}^{*k})$ for the system (1) where optimal control policy $\mathbf{u}^{*k}$ minimizes the cost function (6) while the the worst case disturbance $\mathbf{w}^{*k}$ maximizes it. We will find the range of H^∞ disturbance attenuation level γ and control packet arrival rates α and β such that an optimal controller can be designed to generate $\mathbf{u}^{*k}$ which stabilizes the closed loop system.

3. MAIN RESULTS

A. Finite Horizon Case:

We will employ a dynamic programming approach to get the saddle point condition and the corresponding value function of the zero-sum dynamic game (6). Now let us define the cost-to-go for value function from stage k as follows:

$$V_k(x_k) \triangleq \min_{u_k} \max_{w_k} \mathbb{E}\,\Big[||x_k||^2_W + v_k||u_k||^2_R - \gamma^2 ||w_k||^2 + V_{k+1}(x_{k+1})|\mathcal{I}_k\Big] \tag{8}$$

The above equation is also called the Isaaqs equation (finite horizon version).

Theorem 1: For the zero-sum game (6),

(i) The value function $V_k(x_k)$ at any stage $k \in [0, N-1]$ can be expressed as,

$$V_k(x_k) = \mathbf{E}(x_k^T S_k x_k | \mathcal{I}_k) \quad if\ v_{k-1} = 0 \tag{9a}$$

$$V_k(x_k) = \mathbf{E}(x_k^T U_k x_k | \mathcal{I}_k) \quad if\ v_{k-1} = 1 \tag{9b}$$