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Robust Stability of Uncertain Discrete-time Linear Systems with Input and Output Quantization

L. Su * and G. Chesi *

* The University of Hong Kong, Pokfulam Road, Hong Kong (e-mail: llsu@eee.hku.hk chesi@eee.hku.hk)

Abstract: This paper investigates the robust stability of uncertain discrete-time linear systems with both input and output quantization. Specifically, the output of the plant and the output of the dynamic controller are quantized via two independent static logarithmic quantizers. In fact, there are three blocks of uncertainties under consideration due to the double quantization and uncertain plant. First, a necessary and sufficient condition in terms of LMIs is proposed for the quadratic stability of the closed-loop system with double quantization and norm bounded uncertainty in the plant. Moreover, it is shown that the proposed condition can be exploited to derive the coarsest logarithmic quantization density under which the uncertain plant can be quadratically stabilized via quantized state feedback. Lastly, a new class of Lyapunov function which depends on the quantization errors in a multilinear way is developed to obtain less conservative results.

Keywords: Robust stability, logarithmic quantizer, input and output quantization, uncertain system, Lyapunov function, LMI.

1. INTRODUCTION

Motivated by finite network resource, quantized feedback control has been one of the most popular research trends in the field of networked control systems. It naturally becomes significant that how the quantization error will influence the stability and performance of the feedback systems. In the meanwhile, a great deal of effort has gone into establishing the minimum feedback information needed to stabilize a open-loop unstable system.

Perhaps the most important result in recent years on the quantized feedback control should be traced back to Elia and Mitter (2001) where the logarithmic quantization was proposed and shown to be the coarsest quantizer to quadratically stabilize discrete-time linear time-invariant systems. The logarithmic quantizer was further investigated by Fu and Xie (2005) in which the sector bound approach was exploited to relate the design problem for quantized feedback control to the optimal H_∞ control problem. The same problem has also been resolved in Gao and Chen (2008) based on the utilization of a quantization dependent Lyapunov function. Besides, the quantized feedback control problem has been studied in different scenarios. For instance, Gu and Qiu (2014) put forward the polar logarithmic quantization for multi-input systems; Gu et al. (2015) studied mean-square stabilization for networked control systems with both fading channels and logarithmic quantization; Coutinho et al. (2010), Xia et al. (2013) considered feedback control systems with both input and output quantization. More recently, another research line that researchers have started to deal with is quantization for uncertain systems. See, e.g., Fu and Xie (2010) where sufficient condition was proposed for robust stabilization for linear uncertain systems via logarithmic quantized feedback, Liu et al. (2015) which studied the stability analysis of continuous-time uncertain system with dynamic quantization and communication delays, Kang and Ishii (2015) which considered coarsest quantization for networked control of a class of finite-order uncertain autoregressive plant.

In this paper, we consider the model of double quantization as studied in Coutinho et al. (2010) with the plant affected by unstructured uncertainty. Specifically, the output of the plant and the output of the dynamic controller are quantized via two independent static logarithmic quantizers. In fact, there are three blocks of uncertainties under consideration due to the double quantization and uncertain plant. First, a necessary and sufficient condition in terms of LMIs is proposed for the quadratic stability of the closed-loop system with double quantization and norm bounded uncertainty in the plant. Moreover, it is shown that the proposed condition can be exploited to derive the coarsest logarithmic quantization density under which the uncertain system can be quadratically stabilized via quantized state feedback. Lastly, a new class of Lyapunov function which depends on the quantization errors in a multilinear way is developed to obtain less conservative results.

2. PROBLEM FORMULATION

In this work, we focus on the robust stability of linear systems with input and output quantization, the model of which is depicted in Figure 1. Let us start by considering the single-input single-output (SISO) plant affected by uncertainty described as

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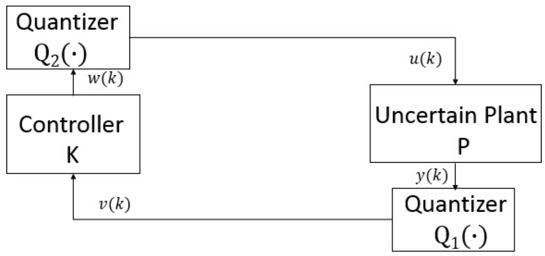

Fig. 1. The uncertain closed-loop system with input and output quantization

$$\begin{cases} x_p(k+1) = (A+A_1)x_p(k) + (B+B_1)u(k) \\ y(k) = Cx_p(k) \end{cases} \quad (1)$$

where $x_p(k) \in \mathbb{R}^n$ is the plant state, $u(k) \in \mathbb{R}$ is the plant input and $y(k) \in \mathbb{R}$ is the plant output, $(A,\ B)$ is the nominal system and the time-varying uncertainty $(A_1,\ B_1)$ is assumed to be norm bounded satisfying

$$\begin{array}{c}[A_1\ B_1] = HF(k)[E_1\ E_2] \\ F(k)F(k)^T \leq I. \end{array} \quad (2)$$

The controller is assumed to be dynamic, described as

$$\begin{cases} x_c(k+1) = A_c x_c(k) + B_c v(k) \\ w(k) = C_c x_c(k) + D_c v(k) \end{cases} \quad (3)$$

where $x_c(k) \in \mathbb{R}^{n_c}$ is the state of controller, $v(k) \in \mathbb{R}$ is the controller input and $w(k) \in \mathbb{R}$ is the controller output.

Following the works Elia and Mitter (2001) and Fu and Xie (2005), we utilize the logarithmic quantization defined as

$$Q(v) = \begin{cases} \rho^i & \text{if } \dfrac{1}{1+\delta}\rho^i < v \leq \dfrac{1}{1-\delta}\rho^i \\ & v > 0,\ i = \pm 1, \pm 2, \ldots \\ 0 & \text{if } v = 0 \\ -Q(v) & \text{if } v < 0 \end{cases} \quad (4)$$

where $0 < \rho < 1$ is the quantization density and $\delta = \frac{1-\rho}{1+\rho}$.

It is assumed that the output of the plant $y(k)$ is quantized before being sent to the input of the controller $v(k)$ and the the output of the controller $w(k)$ is quantized before being sent to the input of the plant $u(k)$. The two quantizers are modeled as

$$\begin{cases} v(k) = Q_1(y(k)) \\ u(k) = Q_2(w(k)) \end{cases} \quad (5)$$

where $Q_1(\cdot)$ and $Q_2(\cdot)$ are static logarithmic quantizers with quantization density ρ_1 and ρ_2.

Let $x(k) = [x_p(k)^T\ x_c(k)^T]^T$ be the state of the closed-loop system. Comprising the plant, the controller and the quantizers, such a closed-loop system is given by

$$x(k+1) = \begin{pmatrix} x_p(k+1) \\ x_c(k+1) \end{pmatrix} = \begin{pmatrix} (A+A_1)x_p(k) \\ A_c x_c(k) \end{pmatrix} + \begin{pmatrix} (B+B_1)Q_2(C_c x_c(k) + D_c Q_1(Cx_p(k))) \\ B_c Q_1(Cx_p(k)) \end{pmatrix}. \quad (6)$$

The problem addressed in this paper is stated as follows.

Problem 1. Establish whether the closed-loop system (6) is robustly stable for all admissible uncertainty $[A_1\ B_1]$ defined in (2).

3. MAIN RESULTS

3.1 Robust quadratic stability

When there is no uncertainty in the plant, i.e., $A_1 = 0$ and $B_1 = 0$, it is shown in Theorem 2 of Coutinho et al. (2010) that the closed-loop system (6) is quadratically stable if and only if there exists a symmetric constant matrix $P > 0$ such that

$$\begin{array}{c}\bar{A}(\Delta_1,\Delta_2)^T P \bar{A}(\Delta_1,\Delta_2) - P < 0 \\ \forall|\Delta_1| \leq \delta_1, |\Delta_2| \leq \delta_2 \end{array} \quad (7)$$

where

$$\bar{A}(\Delta_1,\Delta_2) = \begin{pmatrix} A + B(1+\Delta_2)D_c(1+\Delta_1)C & B(1+\Delta_2)C_c \\ B_c(1+\Delta_1)C & A_c \end{pmatrix}. \quad (8)$$

Thus, from the perspective of quadratic stability, it is not conservative to treat the quantization errors as sector bounded time-varying uncertainties.

Lemma 2. (Amato (2006), Garofalo et al. (1993)) The condition in (7)-(8) holds if and only if there exists a symmetric matrix $P > 0$ such that the following LMIs hold:

$$\begin{array}{c}\begin{pmatrix} P & \bar{A}(\Delta_1,\Delta_2)^T P \\ * & P \end{pmatrix} > 0 \\ \forall \Delta_1 \in \{-\delta_1, \delta_1\} \\ \forall \Delta_2 \in \{-\delta_2, \delta_2\}. \end{array} \quad (9)$$

Lemma 2 has established the equivalence between the quadratic stability of an uncertain system depending multi-affinely on uncertain parameters constrained into a hyper-rectangle and stability of the systems with uncertainty fixed at the vertices under the same Lyapunov function $v(x(k)) = x(k)^T P x(k)$. Lemma 2 will be invoked in the subsequent results.

Next, let us take the uncertainty (A_1, B_1) into consideration. Define the auxiliary system for (6) as

$$\begin{cases} x(k+1) = \hat{A}(\Delta_1(k),\Delta_2(k))x(k) \\ \hat{A}(\Delta_1,\Delta_2) = \begin{pmatrix} A+A_1 & 0 \\ 0 & A_c \end{pmatrix} + \\ \quad \begin{pmatrix} (B+B_1)(1+\Delta_2)([0\ C_c] + D_c(1+\Delta_1)[C\ 0]) \\ B_c(1+\Delta_1)[C\ 0] \end{pmatrix} \\ \forall|\Delta_1(k)| \leq \delta_1, |\Delta_2(k)| \leq \delta_2. \end{cases} \quad (10)$$

Before proceeding to our main result, let us report the following result (see, e.g., Xie (1996)).

Lemma 3. Given real matrices $\mathcal{S} = \mathcal{S}^T, \mathcal{U}, \mathcal{V}$ with appropriate dimension, then

$$\mathcal{S} + \mathcal{U}F(k)\mathcal{V} + \mathcal{V}^T F(k)^T \mathcal{U}^T > 0 \quad (11)$$

holds for all $F(k)$ satisfying $F(k)F(k)^T \leq I$ if and only if there exists a scalar $\sigma > 0$ such that

$$\mathcal{S} - \sigma\mathcal{U}\mathcal{U}^T - \sigma^{-1}\mathcal{V}^T\mathcal{V} > 0. \quad (12)$$