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Identification of Fractional Differential Models for Lithium-ion Polymer Battery Dynamics

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Abstract: This paper presents a non-integer order (fractional) derivative model for modeling lithium-ion (Li-ion) battery dynamics in which direct continuous-time (CT) model identification methods are used to estimate the battery model parameters. In particular, the CT least squares-based state-variable filter (*lssvf*) identification method is extended to be used in fractional differential model identification of a battery system. The model performance and validation accuracy are evaluated on the basis of experimental data of a Li-ion polymer (LiPo) battery. It is shown how the proposed *lssvf* identification method can accurately capture the fractional order battery dynamics and exhibit better performance than integer order differential models.

Keywords: Fractional model; System identification; Battery management system (BMS); Least squares-based state-variable filter (LSSVF) method.

1. INTRODUCTION

Lithium-ion (Li-ion) batteries are currently one of the leading candidates for energy storage and deliver applications in electric vehicles (EVs) and hybrid electric vehicles (HEVs), because they provide high energy density, exhibit very low memory effect, and have typically long cycle-life (Armand and Tarascon, 2008). In order to maintain battery performance and ensure battery operation, a battery management system (BMS) is used to monitor the status of the battery, including state of charge (SOC), state of health (SOH), and state of power (SOP) (Fang et al., 2014; Xia and Mi, 2016). High overcharge/disovercharge rates can damage Li-ion batteries and signal processing and diagonistic algorithms are needed in a BMS to extent longevity, calculate additional maintenance parameters, and control the operating environment of the battery.

A detailed model for Li-ion batteries can be used in a BMS to predict internal battery temperature, the SOC within the individual electrodes, overpotential, and current distribution across the electrodes. Among different model approaches, a BMS may use an electrochemical model that is usually more accurate to capture battery dynamics than an equivalent circuit model (ECM). Alternatively, the battery model may be based simply on the input/output behavior describing power storage and delivery dynamics (Jiang et al., 2016; Zhao and de Callafon, 2016). An electrochemical model has the advantage of predicting various physical information in a Li-ion battery. However, the partial differential equations (PDEs) used in an electrochemical model may require extensive numerical computations to compute a simulation. Although a single particle model (SPM) can be used to simplify the ECM, parameter estimation and model validation are still a challenging task when the only battery states can be measured by voltage, current, temperature, and electrolyte ion concentration information (Guo et al., 2011).

Another way to include model-based signal processing in a BMS is the use of an ECM that is composed of equivalent potential, internal resistance, and effective capacitance (Xia et al., 2016). Although this model has a simplified structure and less parameters to be estimated, it is typically unable to match electrochemical impedance spectroscopy (EIS) experimental data due to its linearity and finite order model structure (He et al., 2011). An alternative approach to a linear time varying ECM is to use fractional derivatives instead of integer derivatives to describe infinite dimensional battery dynamics behavior, and balance the model accuracy and complexity (Zou et al., 2016). Fractional models are an important tool in modeling thermal diffusion and electrochemical diffusion of viscoelastic materials (Malti, 2006). Unfortunately, recently developed parameter estimation methods in fractional derivative models for battery systems may be computationally intensive due to integration and convolution computations (Eckert et al., 2015).

This paper builds on the ideas of fractional model identification of battery systems presented in Eckert et al. (2015), but the problem is formulated as a continuous-time (CT) system identification approach (Garnier and Wang, 2008). The direct identification of CT models is

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shown to work well on stiff systems that have a large range in dynamic behavior as typically seen in a Li-ion battery system. In CT identification, state-variable filters (SVFs) are utilized to filter (smoothen) fractional derivative terms of input/output signals for the estimation of parameters. After obtaining fractional derivative filtered input/output signals, a least squares-based state-variable filter (*lssvf*) of Malti et al. (2008) is used to estimate the parameters of the proposed fractional differential battery model.

2. FRACTIONAL DIFFERENTIAL SYSTEMS

2.1 General linear fractional differential system

A fractional differential equation is usually used to represent a linear fractional model of the format

$$\begin{aligned} &y(t) + a_1 \mathrm{D}^{\alpha_1} y(t) + ... + a_n \mathrm{D}^{\alpha_n} y(t) \\ &= b_0 \mathrm{D}^{\beta_0} u(t) + b_1 \mathrm{D}^{\beta_1} u(t) + ... + b_m \mathrm{D}^{\beta_m} u(t) \end{aligned} \tag{1}$$

where $(a_j, b_i) \in \mathrm{R}^2$, differentiation orders $\alpha_1 < \alpha_2 < ... < \alpha_n$, $\beta_0 < \beta_1 < ... < \beta_m$, and $\alpha_i, \beta_i \in \mathrm{R}^+$. The α-th fractional order fundamental operator is defined according to Petr (2011) via $\mathrm{D}^\alpha = (\frac{d}{dt})^\alpha, \ \forall \alpha \in \mathrm{R}^+$ where the α-th order uninitialized fractional derivative of a function $f(t)$ is given by

$$\mathrm{D}^\alpha f(t) = (\frac{d}{dt})^{\lceil \alpha \rceil} \frac{1}{\Gamma(\lceil \alpha \rceil - \alpha)} \int_0^t \frac{f(\tau)}{(t-\tau)^{\alpha - \lfloor \alpha \rfloor}} d\tau, \tag{2}$$

where Eulers gamma function $\Gamma(\gamma) = \int_0^\infty t^{\gamma-1} \mathrm{e}^{-t} \, \mathrm{d}t$.It is worth noting that ceiling $\lceil \cdot \rceil$ and floor $\lfloor \cdot \rfloor$ functions are equal to the largest previous or the smallest following integer, respectively.

The Laplace transform of the derivative-integral (2) has the form (Podlubny, 1997):

$$\mathcal{L}\{\mathrm{D}^\alpha f(t)\} = s^\alpha F(s), \ \text{ if } f(t) = 0 \ \forall t \leq 0 \tag{3}$$

allowing the fractional differential equation (1) to be written in a transfer function format

$$\mathrm{G}(s) = \frac{b_0 s^{\beta_0} + b_1 s^{\beta_1} + \cdots + b_m s^{\beta_m}}{1 + a_1 s^{\alpha_1} + \cdots + a_n s^{\alpha_n}} \tag{4}$$

where input $u(t)$ and output $y(t)$ signals are assumed to be equal to 0 when $t < 0$. The transfer function representation (4) is adopted for a battery model in this paper.

2.2 Numerical Computation of Fractional Derivatives

In order to compute the system response of an input signal in CT fractional system, the revised Grünwald-Letnikov definition

$$\mathrm{D}^\alpha f(t) = \lim_{\triangle x \to 0} \frac{1}{\triangle x^\alpha} \sum_{j=0}^{[\frac{t-a}{\triangle x}]} (-1)^j \binom{\alpha}{j} f(t - j \triangle x) \tag{5}$$

is used here to derive α-th fractional derivative of a function $f(t)$. The numerical solution to the above revised Grünwald-Letnikov of a general fractional differential equation in (1) can be obtained by a recursive method

$$y(t) = \frac{1}{\sum_{i=0}^n \frac{a_i}{\triangle x^{\alpha_i}}} \left(u(t) - \sum_{i=0}^n \frac{a_i}{\triangle x^{\alpha_i}} \sum_{j=1}^{[\frac{t-a}{\triangle}]} \mu_j^{(\alpha_j)} y_{t-j\triangle x} \right) \tag{6}$$

as done in Tepljakov et al. (2011); Chen et al. (2009), where $\triangle x$ is computation step size. In the above equation (6), the $\mu_j^{(\cdot)}$ can be computed recursively via $\mu_0^\alpha = 1$ and $\mu_j^\alpha = (1 - \frac{\alpha+1}{j})\mu_{j-1}^\alpha, j = 1, 2, \ldots$ to get numerical fractional derivatives of input/output signals. Finally, $\hat{u}(t)$ can be numerically computed by using revised Grünwald-Letnikov (5) to replace the $(-1)^\alpha \binom{\alpha}{j} = \mu_j^\alpha$ term in the computation. As a final note it should be mentioned that this recursive method is actually a fixed-step computation method. As a result, it is necessary to take care of step-size $\triangle x$ in order to increase the computation accuracy.

2.3 Fractional battery model

In this paper, the fractional battery model of Eckert et al. (2015) with a single "RQ element" is used. In particular, the fractional battery model is composed by an inner resistance R_0, a differential capacity C_0, and a RQ element. The model can be described by first (fractional) order transfer function

$$u_0 = \frac{b_3 s^{\alpha+1} + b_2 s + b_1 s^\alpha + b_0}{a_0 s^{\alpha+1} + s} i, \tag{7}$$

where $b_0 = \frac{1}{C_0}$, $b_1 = \frac{RQ}{C_0}$, $b_2 = R + R_0$, $b_3 = R_0 RQ$, $a_0 = RQ$, and $\alpha \in \mathrm{R}^+$. The differential capacity C_0 is assumed to be known and removed, allowing the model to be reduced via $\tilde{u}_t = u_0 - u_{C_0}$ into

$$\frac{b_3 s^\alpha + b_2}{a_0 s^\alpha + 1} = \frac{R_0 RQ s^\alpha + (R + R_0)}{RQ s^\alpha + 1} \tag{8}$$

where the fractional model order α can be recognized on the Laplace variable s. This model will be used as a basis for the parametrization and identification of the battery model parameters.

3. ALGEBRAIC IDENTIFICATION OF FRACTIONAL BATTERY MODEL

3.1 Algebraic identification method

The parameters R_0, R, α and Q in (8) can be estimated via the algebraic identification method by the following approach (Eckert et al., 2015). The approach in Eckert et al. (2015) defines the reparametrization

$$P(t) \cdot \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{bmatrix} = Q(t), \tag{9}$$

where the parameters

$$\begin{aligned} \theta_1 &= -(R + R_0), \\ \theta_2 &= 2R_0 + R, \\ \theta_3 &= \frac{-R}{\alpha}, \end{aligned} \tag{10}$$

can be computed if the matrix