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Change detection and isolation in mechanical system parameters based on perturbation analysis

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Abstract: The monitoring of mechanical systems aims at detecting damages at an early stage, in general by using output-only vibration measurements under ambient excitation. In this paper, a method is proposed for the detection and isolation of small changes in the physical parameters of a linear mechanical system. Based on a recent work where the multiplicative change detection problem is transformed to an additive one by means of perturbation analysis, changes in the eigenvalues and eigenvectors of the mechanical system are considered in the first step. In a second step, these changes are related to physical parameters of the mechanical system. Finally, another transformation further simplifies the detection and isolation problem into the framework of a linear regression subject to additive white Gaussian noises, leading to a numerically efficient solution of the considered problems. A numerical example of a simulated mechanical structure is reported for damage detection and localization.

Keywords: Fault detection and isolation, statistical tests, mechanical system, vibrations.

1. INTRODUCTION

The detection and localization of damages based on measured vibration data are fundamental tasks for structural health monitoring to allow an automated damage diagnosis [Farrar and Worden, 2007]. Early sign of damages can be modeled as changes in the parameters of the underlying mechanical system. They affect the dynamic properties of a structure, inducing *small changes* in the eigenstructure (eigenvalues and eigenvectors) of a linear system. It is of interest to *detect* these changes and to decide which of the physical parameters are responsible for the change (*isolation*). A particular difficulty for structural health monitoring is caused by the absence of known system inputs, since the structural excitation is usually only ambient, leading to an output-only monitoring problem.

Among the many model-based or data-based methods for mechanical structure damage detection [Carden and Fanning, 2004], methods based on direct model-data matching are particularly appealing for an automated damage diagnosis, where current measurement data are directly confronted to a reference model. For instance, such methods include non-parametric change detection based on novelty detection [Worden et al., 2000] or whiteness tests on Kalman filter innovations [Bernal, 2013]. Another method within this category, the local asymptotic approach to change detection [Benveniste et al., 1987], has the ability of focusing the detection on some chosen system parameters. Associated to efficient hypothesis testing tools, this method has led to successful applications in the field of vibration monitoring, e.g. in [Jhinaoui et al., 2012, Döhler and Mevel, 2013, Döhler et al., 2014], including fault isolation and estimation [Döhler et al., 2016].

In [Döhler et al., 2015] an alternative method for parametric change detection has been developed, where the *multiplicative* eigenstructure change detection problem is transformed to an *additive* one by means of a perturbation analysis, assuming small parameter changes. Amongst others, this allows addressing random uncertainties more efficiently in associated hypothesis testing tools by avoiding the covariance matrix estimation problem encountered in the local asymptotic approach. In the current paper, we extend this approach from *eigenstructure parameter change detection* to *mechanical system parameter change detection*. By linking changes in the system matrices of a state-space model to mechanical system parameters, the underlying physical problem of *fault isolation*, i.e. of deciding which physical parameters are responsible for the detected changes, is solved.

This paper is organized as follows. In Section 2, the system models and parameters are defined. In Section 3, the perturbation analysis is carried out to transform the system parameter change detection problem into an additive one. In Sections 4 and 5 the respective hypothesis test for change detection and isolation are stated. Finally, an application for vibration-based damage detection and localization is shown in Section 6.

2. PROBLEM STATEMENT

The behavior of mechanical structures subject to unknown ambient excitation can be described by the differential equation

$$\mathcal{M}\ddot{\mathcal{X}}(t) + \mathcal{C}\dot{\mathcal{X}}(t) + \mathcal{K}\mathcal{X}(t) = f(t) \tag{1}$$

where t denotes continuous time; $\mathcal{M}, \mathcal{C}, \mathcal{K} \in \mathbb{R}^{m\times m}$ are mass, damping, and stiffness matrices, respectively; the

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state vector $\mathcal{X}(t) \in \mathbb{R}^m$ is the displacement vector of the m degrees of freedom of the structure; and $f(t)$ is the external unmeasured force (random disturbance).

Observed at r sensor positions by displacement, velocity or acceleration sensors at discrete time instants $t = k\tau$ (with sampling rate $1/\tau$), system (1) can also be described by a discrete-time state space system model [Juang, 1994]

$$\begin{cases} z_{k+1} = Fz_k + w_k \\ y_k = Hz_k + v_k \end{cases} \tag{2}$$

where the state vector $z_k = \begin{bmatrix} \mathcal{X}(k\tau)^T & \dot{\mathcal{X}}(k\tau)^T \end{bmatrix}^T \in \mathbb{R}^n$ with $n = 2m$, the measured output vector $y_k \in \mathbb{R}^r$ and the system matrices

$$F = \exp(F^c\tau) \in \mathbb{R}^{n\times n}, \quad F^c = \begin{bmatrix} 0 & I \\ -\mathcal{M}^{-1}\mathcal{K} & -\mathcal{M}^{-1}\mathcal{C} \end{bmatrix}, \tag{3}$$

$$H = \begin{bmatrix} L_d - L_a\mathcal{M}^{-1}\mathcal{K} & L_v - L_a\mathcal{M}^{-1}\mathcal{C} \end{bmatrix} \in \mathbb{R}^{r\times n}, \tag{4}$$

with selection matrices $L_d, L_v, L_a \in \{0,1\}^{r\times m}$ indicating the positions of displacement, velocity or acceleration sensors, respectively. The state noise w_k and output noise v_k are unmeasured and assumed to be Gaussian, zero-mean and white.

In this paper, damages are considered as changes in the structural stiffness properties of system (1), corresponding to changes related to the parameters of structural elements. The corresponding stiffness matrix $\mathcal{K}$ can be parametrized by a vector of independent parameters η with $\mathcal{K} = \mathcal{K}(\eta)$. No changes are assumed in $\mathcal{M}$ and $\mathcal{C}$. Changes in the physical parameter η provoke changes in the eigenstructure of system (1), and consequently of system (2). The related eigenstructure parameter vector θ is defined in the following.

The eigenvalues μ_i and eigenvectors ϕ_i of system (1) satisfy

$$(\mathcal{M}\mu_i^2 + \mathcal{C}\mu_i + \mathcal{K})\phi_i = 0, \quad i = 1, 2, \ldots, 2m.$$

They are related to the eigenvalues and eigenvectors of the matrix F in (2), which satisfy

$$F\psi_i = \lambda_i\psi_i, \quad i = 1, 2, \ldots, n = 2m, \tag{5}$$

through

$$\lambda_i = \mathrm{e}^{\mu_i\tau}, \quad \psi_i = \begin{bmatrix} \phi_i \\ \mu_i\phi_i \end{bmatrix}. \tag{6}$$

Assume that the eigenstructure of the considered system contains only complex modes. This is the typical case for structural health monitoring applications. Then, the eigenvalues μ_i consist of m conjugate complex pairs, so do the eigenvalues λ_i. Let the vectors

$$\mu \triangleq [\mu_1, \mu_2, \ldots, \mu_m]^T \in \mathbb{C}^m, \quad \lambda \triangleq [\lambda_1, \lambda_2, \ldots, \lambda_m]^T \in \mathbb{C}^m$$

contain m of the n eigenvalues, one out of each of the m conjugate complex pairs, and

$$\phi \triangleq [\phi_1, \phi_2, \ldots, \phi_m] \in \mathbb{C}^{m\times m}$$

be composed of the corresponding eigenvectors. The complex eigenvalues and eigenvectors (μ_i, ϕ_i) are then represented by the equivalent real eigenstructure parameter vector $\theta \in \mathbb{R}^{2m+2m^2}$ defined as

$$\theta \triangleq \begin{bmatrix} \Re(\mu) \\ \Im(\mu) \\ \mathrm{vec}(\Re(\phi)) \\ \mathrm{vec}(\Im(\phi)) \end{bmatrix} \tag{7}$$

where $\Re$ and $\Im$ denote respectively the real part and the imaginary part of a complex variable.

Assume that the matrix F in (5) is diagonalizable, then

$$F = T(\theta)A(\theta)T^{-1}(\theta) \tag{8}$$

with real matrices

$$A(\theta) = \begin{bmatrix} \mathrm{diag}(\Re(\lambda)) & \mathrm{diag}(\Im(\lambda)) \\ -\mathrm{diag}(\Im(\lambda)) & \mathrm{diag}(\Re(\lambda)) \end{bmatrix}, \tag{9}$$

$$T(\theta) = \begin{bmatrix} \Re(\phi) & \Im(\phi) \\ \Re(\phi\,\mathrm{diag}(\mu)) & \Im(\phi\,\mathrm{diag}(\mu)) \end{bmatrix}. \tag{10}$$

Similarly, following from (3), (4) and (8), matrix H yields

$$H = [L_d \ \ L_v] + [0_{r,m} \ \ L_a]\, T(\theta)A_c(\theta)T^{-1}(\theta) \tag{11}$$

where

$$A_c(\theta) = \begin{bmatrix} \mathrm{diag}(\Re(\mu)) & \mathrm{diag}(\Im(\mu)) \\ -\mathrm{diag}(\Im(\mu)) & \mathrm{diag}(\Re(\mu)) \end{bmatrix}.$$

With the parametrization of F and H with θ expressed in (8) and (11), the state-space model (2) is rewritten as

$$z_{k+1} = T(\theta)A(\theta)T^{-1}(\theta)z_k + w_k \tag{12a}$$

$$y_k = \left([L_d \ \ L_v] + [0_{r,m} \ \ L_a]\, T(\theta)A_c(\theta)T^{-1}(\theta)\right) z_k + v_k. \tag{12b}$$

In this paper, it is assumed that the nominal values of the mechanical system matrices $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ are available, typically based on a finite element model of the monitored structure. The nominal value θ^0 of the parameter vector θ is then accordingly deduced. Note that the estimation of $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ from output-only sensor data is in general not possible. While the estimation of F and H would be possible, e.g. by subspace system identification [Van Overschee and De Moor, 1996], they can only be estimated up to an unknown similarity transformation, and it is not possible to fully deduce the parameter vector θ from such a result.

Th choice of the parametrization θ is different than in [Döhler et al., 2015], where the eigenvector parts only at the sensor coordinates were part of the parametrization, computed by $H\psi_i$. Though the nominal parameter in [Döhler et al., 2015] can be obtained entirely from measurements without the knowledge of the structural system matrices $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$, it allows only for change *detection*, while no link to the physical properties of the structure was given for fault isolation. In the current paper, a link from θ to the physical parameter set η will be made, which allows for the *isolation* of the physical parameter subset that is responsible for the detected change. Note that this link is necessary for fault isolation, since damages usually correspond to changes in few components of η, whereas in general all components of θ are affected.

Furthermore, monitoring the system in the state basis related to the structural system matrices $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ in (2)–(4) and parameterized in (12) avoids the problem that noise properties are modified by damages when the system is monitored in the modal basis as in [Döhler et al., 2015]. In fact, the state noise term in the modal basis is $T^{-1}(\theta)w_k$ and thus affected by changes in θ, and this dependence in θ was neglected in [Döhler et al., 2015]. In the current paper, change detection and isolation are based on the state-space model (12), formulated in a particular state basis such that the state noise covariance is independent of θ.

In the following, the change detection in θ will be carried out based on a perturbation analysis. Then, a link to the physical parameterization η will be made, and fault isolation will be presented.