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An Unstructured Flexible Nonlinear Model for the Cascaded Water-tanks Benchmark ⋆

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Abstract: Many real world systems exhibit a quasi linear or weakly nonlinear behavior during normal operation, and a hard saturation effect for high peaks of the input signal. A typical example of such systems is the cascaded water-tanks benchmark. This benchmark combines soft and hard nonlinearities to be identified based on relatively short data records. In this paper, a methodology to identify an unstructured flexible nonlinear state space model (NLSS) for the cascaded water-tanks benchmark is proposed. The flexibility of the NLSS model structure is demonstrated by introducing two different initialisation schemes. Furthermore the strengths and short-comings of the model structure are discussed with respect to the cascaded water-benchmark identification problem.

Keywords: System identification; Nonlinear state space model; Short-data records; Soft and hard nonlinearities; Nonlinear systems.

1. INTRODUCTION

There is an evident need of good system modelling techniques in many branches of engineering. Mathematical (linear or nonlinear) models are needed in various applications, for example, to understand and analyse the system under test, to simulate or predict the behavior of the system during the design phase or to design and implement a controller. System identification provides us with a variety of methods to derive accurate mathematical descriptions of the underlying system, based on a set of input/output measurements.

1.1 *Nonlinear System Identification*

The recent years have witnessed the shift from linear system identification (Ljung (1998); Pintelon and Schoukens (2012); Van Overschee and De Moor (2012)) to nonlinear system identification methods, driven by the need to capture the inherent nonlinear effects of real-life systems. Nonlinear system identification constantly faces the challenge of deciding between the flexibility of the fitted model and its parsimony. Flexibility refers to the ability of the model to capture complex nonlinearities, while parsimony is its ability to possess a low number of parameters. A general framework for nonlinear system identification does not exist (Giannakis and Serpedin (2001)), however, modeling nonlinear systems is covered in different fields like statistical learning and machine learning (Suykens et al. (2002); Rasmussen and Williams (2006); Hastie et al. (2009); Suykens et al. (2012)), but most of these methods are typically not specifically developed to deal with dynamics and often have limited means for dealing with noise. Within the system identification community two major approaches to nonlinear system identification can be distinguished: black-box nonlinear system identification (Sjöberg et al. (1995), Billings (2013)) and block-oriented system identification (Giri and Bai (2010), Mzyk (2013)).

State-space models are general representations that allow one to describe a variety of systems. In particular, nonlinear state-space modeling represents a promising, and at the same time challenging, class of techniques. In this paper, we focus mainly on black-box identification of nonlinear state space model (NLSS) structures (Paduart et al. (2010); Schön et al. (2011)). The focus of this paper is the application of two initialization schemes for the identification of nonlinear state-space models for the cascaded water-tanks benchmark problem (Schoukens et al. (2016)) and test their suitability and performance to capture the dynamical behavior of the cascaded water-tanks benchmark problem.

This paper is organized as follows: Section 2 introduces the cascaded water-tanks benchmark and the identification challenges associated with this benchmark problem briefly. Section 3 describes the nonlinear modelling approach using the NLSS model structures used in this paper. The identification of NLSS model along with two different initialisation schemes is described in Section 4 and Section 5 respectively. Section 6 gives provides an overview of the final objective functions, which are minimised using two different initialisation schemes. Section 7 gives an introduction to the experimental set-up as well as the measurement methodology used for the acquisition of the

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signals. Results are presented in Section 8, and finally, the conclusions are given in Section 9.

2. CASCADED WATER TANKS SYSTEM

In this section, we introduce very briefly the cascaded water-tanks system benchmark problem and state the nonlinear identification challenges associated with it.

2.1 System

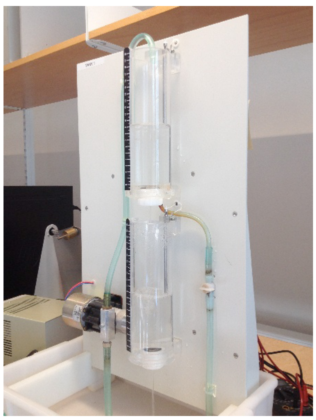

Fig. 1. The water is pumped from a reservoir in the upper tank, flows to the lower tank and finally flows back into the reservoir. The input is the pump voltage, the output is the water level of the lower tank.

The cascaded tanks system is a liquid level control system consisting of two tanks with free outlets fed by a pump. The input signal controls a water pump that pumps the water from a reservoir (considered here as an ideal reservoir which is able to provide enough water) into the upper water tank. The water of the upper water tank flows through a small opening into the lower water tank, and finally through a small opening from the lower water tank back into the reservoir. This process is shown in Figure 1.

The relation between (1) the water flowing from the upper tank to the lower tank and (2) the water flowing from the lower tank into the reservoir are weakly nonlinear functions. However, when the amplitude of the input signal is too large, an overflow can happen in the upper tank, and with a delay also in the lower tank. When the upper tank overflows, a part of the water goes into the lower tank, the rest flows directly into the reservoir. This effect is partly stochastic, hence it acts as an input-dependent process noise source. Fig.2 shows the input-

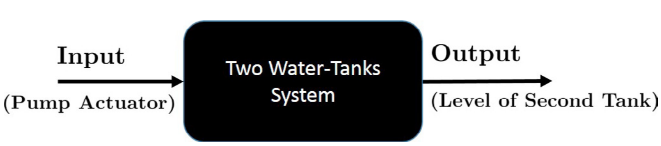

Fig. 2. Block diagram with respective input (pump actuator) and output (height of the second tank) respectively

output block diagram for the water-tanks system shown in Fig.1. Without considering the overflow effect, the following input-output model can be constructed based on Bernoulli's principle and conservation of mass:

$$\dot{x_1}(t) = -k_1\sqrt{x_1(t)} + k_4 u(t) + w_1(t), \tag{1}$$
$$\dot{x_2}(t) = k_2\sqrt{x_1(t)} - k_3\sqrt{x_2(t)} + w_2(t), \tag{2}$$
$$y(t) = x_2(t) + e(t) \tag{3}$$

where $u(t)$ is the input signal, $x_1(t)$ and $x_2(t)$ are the states of the system, $w_1(t)$, $w_2(t)$ and $e(t)$ are the additive noise sources and k_1, k_2, k_3, and k_4 are the constants depending on the system properties. The stochastic nature of the overflow can be captured by the process noise $w_1(t)$, to some extent depending upon the input flowrate.

2.2 Identification Challenges

The major nonlinear system identification challenges associated with the water-tanks benchmark are listed below:

(1) the hard saturation nonlinearity combined with the weakly nonlinear behavior of the system in normal operation,
(2) the overflow from the upper to the lower tank, this effect also introduces input dependent process noise,
(3) the relatively short estimation data record,
(4) the unknown initial values of the states.

In the next section, we introduce the nonlinear state space model structure and discuss two different ways to represent it. Later in the paper, the procedure to identify these two different nonlinear state space model structures from input-output measurements of cascaded water-tanks benchmark will be discussed.

3. NONLINEAR STATE SPACE

Physical interpretation of the system under test is not always required, for instance in control or prediction problems. In that case, the user prefers a flexible and an easy-to-initialize black-box model. Moreover, the model should preferably be able to describe Multiple-Input Multiple-Output (MIMO) systems in a compact way. A good base for such a model is a state space representation of the system under consideration. A general n_x^{th} order discrete-time state space model is described by the following equations:

$$\begin{aligned} x(t+1) &= f(x(t), u(t)) \\ y(t) &= g(x(t), u(t)) \end{aligned} \tag{4}$$

with $u(t) \in \mathbb{R}^{n_u}$ the vector containing the n_u inputs at time t, and $y(t) \in \mathbb{R}^{n_y}$ the vector containing the n_y outputs. The state vector $x(t) \in \mathbb{R}^{n_x}$ represents the memory of the dynamical system. The theoretical analysis for studying the equivalence between the physical continuous time system and this model structure is out of the scope of this paper.

*3.1 **Polynomial Nonlinear State-Space Models***

A nonlinear state space model (where $f(\cdot), g(\cdot)$ are approximated by polynomial basis functions) is termed as a Polynomial Nonlinear State-Space (PNLSS). The PNLSS model structure (Paduart et al. (2010)) is described as:

$$\begin{aligned} x(t+1) &= Ax(t) + Bu(t) + E\zeta(t) \\ y(t) &= Cx(t) + Du(t) + F\eta(t) + e(t) \end{aligned} \tag{5}$$

The coefficients of the linear terms in $x(t) \in \mathbb{R}^{n_x}$ and $u(t) \in \mathbb{R}^{n_u}$ are given by the matrices $A \in \mathbb{R}^{n_x \times n_x}$ and