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Continuous-Time Nonlinear Systems Identification with Output Error Method Based on Derivative-Free Optimisation

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Abstract: The purpose of this study is to tackle the nonlinear system identification benchmarks proposed by (Schoukens and Noël, 2016). Two of the three benchmarks are considered, namely the cascaded tanks setup and the Bouc-Wen hysteretic system. Our approach is an output error method based on continuous time models. Due to the nonlinearities, the derivatives of the output with respect to the parameters are not defined everywhere. We compare the performance of two derivative-free optimisation solvers: the Nelder-Mead simplex and the NOMAD algorithm. Both are available in the OPTI Toolbox. The results suggest that the method is appropriate for those systems. However, it is not possible to discriminate between both optimisation solvers.

Keywords: Identification and modelling; Continuous time system estimation; Nonlinear system identification; Output error method

1. INTRODUCTION

In robotics and mechanical engineering the dynamic models are based on differential equations which often result from Newton's law or Lagrange equations. Recently, the identification of continuous-time models has grown in popularity in the field of Automatic Control (Garnier and Wang, 2008) and see the recent special issue in the International Journal of Control (Garnier and Young, 2014). The output error method is an option to deal with such problems. It consists in minimizing the difference between the simulated model output and the measured output. This approach has proven its suitability in Automatic Control (Carrillo et al., 2009), in robotics (Gautier et al., 2013) and in aeronautics (Klein and Morelli, 2006) for instance.

The aim of this paper is to evaluate if the continuous-time output error method is suitable for identifying two of the non-linear systems proposed by (Schoukens and Noël, 2016) as benchmarks for the community. We will deal with the parametric identification of the Bouc-Wen hysteretic system and the cascaded tanks setup. As it will be seen, the models are continuous but not differentiable everywhere. Thus, optimisation algorithms based on the differentiability of the cost functions cannot be employed. Two derivative-free algorithms are used and compared: the well-known Nelder-Mead simplex and the recent NOMAD optimizer, which are both available in the free OPTI Toolbox for MATLAB©.

This paper is organised as follows. Section 2 deals with the general methodology for output error identification in continuous time framework and presents the optimisation algorithms considered. In section 3, the model of the cascaded tanks setup is developed and the results are detailed. Section 4 follows the same structure for the Bouc-Wen hysteretic system. Finally, section 5 provides concluding remarks.

2. GENERAL METHODOLOGY

2.1 Continuous Time Output Error Method

With the Output Error Method (OEM), the unknown system parameters are tuned so that the simulated model output fits the measured system output. To evaluate the difference between the two outputs many criteria may be used, as explained in (Walter and Pronzato, 1997). The criterion minimisation is usually solved thanks to non-linear optimisation algorithms based on a first- or second-order Taylor series expansion. That requires the computation of the criterion derivatives with respect to the parameters. In some cases those derivatives can be exactly known, but in most cases the derivatives are approximated by finite differences.

To simulate the continuous-time system and obtain a simulated output, the differential equations must be solved. Many numerical solvers exist in the literature like the well-known Runge-Kutta method, for further examples see (Hairer et al., 1993). In this article, they will be referred as "integration solvers" to avoid confusion with the "optimisation solvers" introduced in the previous paragraph. In practice, the integration solver needs the same input as the real system and a set of values for the parameters to identify. The choice of the integration solver is decisive. For each model, the practitioner must find the integration solver which suits to the system properties. For instance, if the system presents two dynamics whose the characteristic

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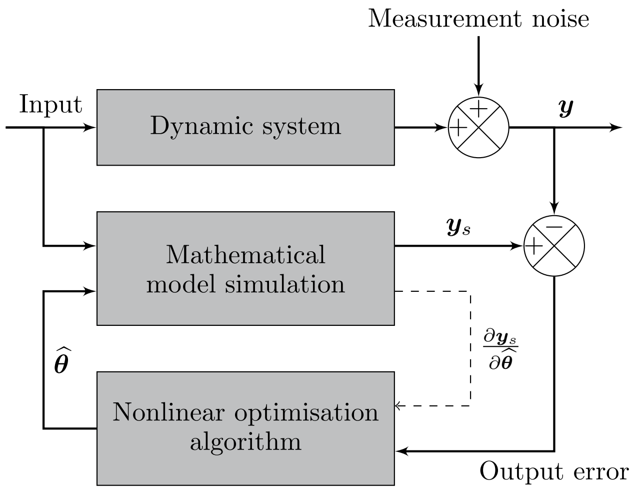

Fig. 1. Output Error Method schematic diagram

times greatly differ, a stiff solver should be employed. If the integration solver is not appropriate, it may lead to a biased identification.

The initial values is a crucial point for OEM. With a bad initialisation the optimisation solver may lead to local minimum (if it is a local optimizer) or even diverge. The integration solver may also diverge if the parameters are not suitable. Depending on the application, different techniques may be used to initialize correctly the method. If the problem is linear with respect to the parameters and if all the states are available, a Least-Squares (LS) estimation can be employed. As shown in (Gautier et al., 2013), in the field of robotics the Computer-Aided Design (CAD) values of the inertia are enough accurate to initialize. In aircraft identification, initial values can be available from wind tunnel test or computational fluid dynamics.

Inspired from (Jategaonkar, 2006), Figure 1 illustrates the OEM principle where $\boldsymbol{y}$ is the $(N_s \times 1)$ vector of the measured output, $\boldsymbol{y_s}$ is the $(N_s \times 1)$ vector of the simulated output, $\widehat{\boldsymbol{\theta}}$ the $(N_\theta \times 1)$ vector of estimated parameters, and $\frac{\partial \boldsymbol{y_s}}{\partial \widehat{\boldsymbol{\theta}}}$ is the output sensitivity, which is a $(N_s \times N_\theta)$ jacobian matrix. N_s is the number of sampling points considered and the N_θ is the number of unknown parameters. As it can be seen, the only stochastic signal is the measurement noise. The input signal is indeed assumed to be noise free. In addition, the integration solver is deterministic. It is consequently impossible to take into account process noise in the simulation. This is why the third proposed benchmark is not considered in this article.

2.2 Optimisation Solvers

OPTI Toolbox This benchmarks challenge was the opportunity to test different optimisation solvers. Our choice was to employ the OPTimization Interface (OPTI) Toolbox developed by (Currie and Wilson, 2012). This toolbox is a free interface between MATLAB© and many open source and academic solvers. From this toolbox, two derivative free algorithms have been selected to solve unconstrained nonlinear least-squares with a quadratic criterion. One noteworthy point is that, after convergence, the toolbox computes the jacobian matrix with finite differences for the statistical analysis.

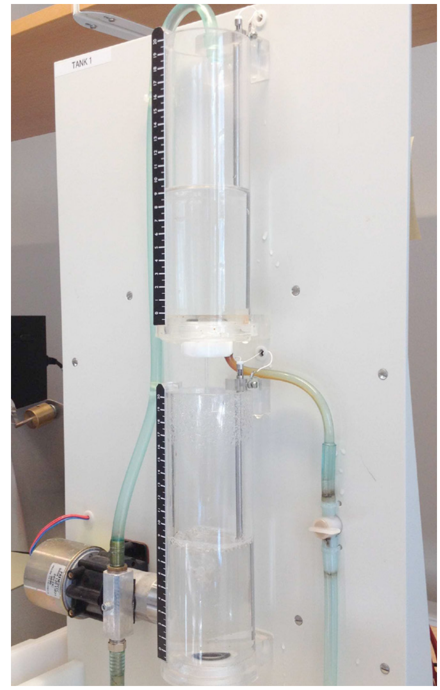

Fig. 2. Cascaded tanks setup

Nelder-Mead Simplex The first considered algorithm is the well-known Nelder-Mead (NM) simplex from (Nelder and Mead, 1965). This heuristic method is based on a polytope of $N_{theta} + 1$ vertices. At each iteration, the vertice, where the cost function is the largest, is modified according to specific rules. It exists many variants of this algorithm depending on the updating rules. We decided to use the algorithm available in the open-source library NLopt developed by (Johnson, 2016) and provided in OPTI.

NOMAD Optimizer The second optimizer is the algorithm called Nonlinear Optimization by Mesh Adaptive Direct (NOMAD) search, from (Abramson et al., 2016). NOMAD is a direct search method, i.e. derivative free. At each iteration, a mesh is designed around the current optimum and the function is evaluated at any mesh points. The mesh is based on a given pattern. Thus, the choice of directions is fixed and finite. If the *search* step does not manage to find a new optimum, it is followed by the *poll* step. This second step consists of a local exploration around the optimum. For this step, the set of points to be evaluated is defined by orthogonal directions which are dense in the unit sphere. At each *poll* step, a new set is constructed. The *search* step is a common element to all Generalized Pattern Search (GPS) algorithms. The specificity of NOMAD lies on the *poll* step which gives more flexibility in the directions.

3. CASCADED TANKS

3.1 Model Description

According to (Schoukens and Noël, 2016), the model of the plant (Fig. 2) comes from Bernoulli's principle and is