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Nonparametric Volterra Series Estimate of the Cascaded Water Tanks Using Multidimensional Regularization

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Abstract: **This paper presents an efficient nonparametric time domain nonlinear system identification method applied to the measurement benchmark data of the cascaded water tanks. In this work a method to estimate efficiently finite Volterra kernels without the need of long records is presented. This work is a novel extension of the regularization methods that have been developed for impulse response estimates of linear time invariant systems. Due to the limited number of available data samples, the highest considered Volterra order is limited. In the paper the results for different scenarios varying from a simple Finite Impulse Response (FIR) model to a 3rd degree Volterra series are compared and studied. In each case, the transients are removed by a special regularization method based on the novel ideas of transient removal for Linear Time-Varying (LTV) systems. Using the proposed methodologies, the nonparametric Volterra models provide a very good data-fit, and their performance is comparable with the white-box (physical) models.**

Keywords: **Bayesian methods, Software for system identification, Filtering and smoothing, Nonparametric methods, Nonlinear system identification, Grey box modelling.**

1. INTRODUCTION

In the field of system identification, modelling of nonlinear systems is one of the most challenging tasks. One possibility to model nonlinear dynamics in a nonparametric way is by means of the Volterra series (Schetzen (1980)). The use of the series can be quite beneficial when precise knowledge about the exact nature of the nonlinear system behaviour is absent. Moreover, it has been shown in Boyd *et al.* (1985) that a truncated version of the infinite Volterra series can approximate the output of any nonlinear system up to any desired accuracy as long as the system is of fading memory. Practically speaking, it is sufficient that the influence of past input signals to the current system output decreases with time, a property that is met quite often in real systems, such as the cascaded water tank system considered in this paper.

However, it is quite often possible that an accurate nonparametric fitting of nonlinear dynamics requires an excessive number of model parameters. Under this condition, either long data records should be available, resulting in computationally heavy optimization problems and long measurements, or the estimated model parameters will suffer from high variance. This is the reason why the series has in general been of limited use and only for cases where short memory lengths and/or low dimensional series for the modelling process were sufficient (e.g. Koh *et al.* (2013)).

In this paper, we present a method to estimate efficiently finite Volterra kernels in the time domain without the need of long measurements. It is based on the regularization methods that have been developed for FIR modelling of Linear Time-Invariant (LTI) systems (Pillonetto *et al.* (2014b)) while results exist also for the case of Frequency Response Function (FRF) estimation (Lataire *et al.* (2016)). In the aforementioned studies, the impulse response coefficients for a LTI system are estimated in an output error setting using prior information during the identification step in a Bayesian framework. The knowledge available a priori for the FIR coefficients was related to the fact that the impulse response function (IRF) of a stable LTI system is exponentially decaying and moreover, there is a certain level of correlation between the impulse coefficients (smoothness of estimated response).

The regularization methods introduced for FIR modelling are extended to the case of Volterra kernels estimation using the method proposed in Birpoutsoukis *et al.* (2016). The benefit of regularization in this case with respect to FIR modelling is even more evident given the larger number of parameters usually involved in the Volterra series. Prior information about the Volterra kernels includes the decaying of the kernels as well as the correlation between the coefficients in multiple dimensions.

In this work the regularized Volterra kernel estimation technique is combined with a method for transient elimination which plays a key role in this particular benchmark problem because each measurement contains transient. Due to the fact that the measurement length is comparable to the number of parameters, it is necessary to eliminate the undesired effects of the transient as much as possible. The proposed elimination technique uses a special LTI regularization based on the ideas of an earlier work on nonparametric modelling of LTV systems (Csurcsia (2015)).

The paper is organized as follows: in Section 2, the benchmark problem is formulated. Section 3 introduces the regularized Volterra kernel estimation method. In Section 4 the proposed method for the transient removal is presented. Section 5 shows the concrete benchmarks results showing the efficiency of the combination of the two proposed methods for modelling of the cascaded water tank system. Finally, the conclusions are provided in Section 6.

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2. PROBLEM FORMULATION

In this section a brief overview of the benchmark problem is presented. The observed system and its measurements were provided, so the authors and other benchmark performers had no influence on the selection of the measurements. A detailed description of the underlying system, its measurements together with illustrative photos and videos can be found on the website of the workshop (Schoukens *et al*. (2016)).

2.1 Description of the cascaded water tanks

The observed system consists of two vertically cascaded water tanks with free outlets fed by the (input) voltage controlled pump. The water is fed from a reservoir into the upper water tank which flows through a small opening into the lower water tank, where the water level is measured. Finally, the water flows through a small opening from the lower water tank back into the reservoir. The whole process is illustrated in Fig. 1.

Under normal operating conditions the water flows from the upper tank to the lower tank, and from the lower tank back into the reservoir. This kind of flow is weakly nonlinear (Schoukens *et al.* (2016)). However, when the excitation signal is too large for certain duration of time, an overflow (saturation) can happen in the upper tank, and with a delay also in the lower tank. When the upper tank over flows, part of the water goes into the lower tank, the rest flows directly into the reservoir. This kind of saturation is strongly nonlinear.

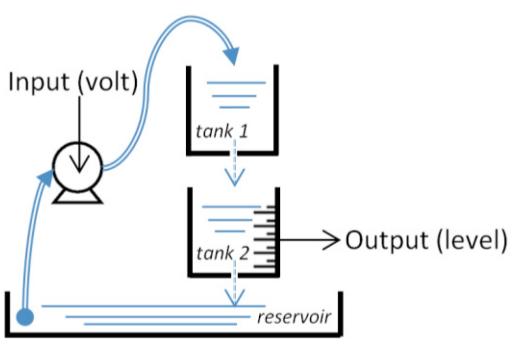

Fig. 1. Schematic of the underlying system: the water is pumped from a reservoir in the uppertank, flows to the lower tank and finally flows back into the reservoir. The input is the pump voltage, the output is the water level of the lower tank.

2.2 Description of measurement

The excitation signal is a random phase multisine (Pintelon et al (2012)) with a length (N) of 1024. The sampling frequency is 0.25 Hz. The excited frequency ranges from 0 to 0.0144 Hz. The Signal-to-Noise Ratio (SNR) is around 40 dB. There are two datasets available for model estimation and validation, respectively. Each measurement starts in an unknown initial state. This unknown state is approximately the same for both the estimation and validation datasets. Furthermore, each dataset contains two overflows at two different time instances.

The water level is measured using uncalibrated capacitive water level sensors which are considered to be part of the system.

2.3 The goal

This work aims at obtaining a nonparametric time domain model based on the estimation data. The goodness of the model fit is measured via the validation data on which the RMS error (see (9)) is calculated. Showing the error levels together with a study will allow potential users to fairly compare different methods, and to motivate them to consider using the proposed method.

3. THE NONPARAMETRIC IDENTIFICATION METHOD

3.1 The model structure

It is assumed that the true underlying nonlinear system can be described by the following finite discrete-time Volterra series:

$$y_{meas}(n) = h_0 + \sum_{\tau_1=0}^{n_m-1} \dots \sum_{\tau_m=0}^{n_m-1} h_m(\tau_1, \dots, \tau_m) \prod_{\tau=\tau_1}^{\tau_m} u(n-\tau) + e(n) \qquad (1)$$

where $u(n)$ denotes the input, $y_{meas}(n)$ represents the measured output signal, $e(n)$ is zero mean i.i.d. white noise with finite variance, $h_m(\tau_1, \dots, \tau_m)$ is the Volterra kernel of order m, $\tau_i, i = 1, \dots, m$ denote the lag variables and $n_m - 1$ corresponds to the memory of h_m. The Volterra kernels are considered to be symmetric, which for the second order kernel is translated to $h_2(\tau_1, \tau_2) = h_2(\tau_1, \tau_2)$, $\forall \tau_1, \tau_2$. Due to symmetry, it can be easily shown that the number of coefficients to be estimated for a Volterra kernel of order $m \geq 1$ is $n_{h_m} = (\frac{1}{m!}) \prod_{i=0}^{m-1}(n_m - i)$. It is also important to clarify the difference between order and degree of the Volterra series with an example: the third degree Volterra series contains the Volterra kernels of order 0, 1, 2 and 3.

3.2 The cost function

Equation (1) can be rewritten into a vectorial form as $\mathrm{Y}_{meas} = \mathrm{K}\theta + \mathrm{E}$, where $\theta \in \mathbb{R}^{n_\theta}$, $n_\theta = 1 + \sum_{m=1}^{M} n_{h_m}$, contains the Volterra coefficients h_m, $\mathrm{K} \in \mathbb{R}^{N \times n_\theta}$ is the observation matrix (the regressor), $\mathrm{Y}_{meas} \in \mathbb{R}^N$ contains the measured output and $\mathrm{E} \in \mathbb{R}^N$ contains the measurement noise.

In this work, the kernel coefficients are estimated by minimizing the following cost function (regularized least squares):

$$\hat{\theta}_{\mathrm{reg}} = \arg\min_\theta \left|\left|\mathrm{Y}_{meas} - \mathrm{K}\,\theta\right|\right|_2^2 + \theta^{\mathrm{T}}\,\mathrm{D}\,\theta \qquad (2)$$

where the block-diagonal matrix $\mathrm{D} \in \mathbb{R}^{n_\theta \times n_\theta}$ contains $(M+1)$ submatrices penalizing the coefficients of the Volterra kernels. It can be observed that the Maximum Likelihood (ML) estimation of the Volterra coefficients can be computed by setting $D = 0$. The ML estimates suffer very often from high variance due to the large number of parameters in the model (curse of dimensionality).