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Analytical Initialization of a Continuation-Based Indirect Method for Optimal Control of Endo-Atmospheric Launch Vehicle Systems

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Abstract: In this paper, we propose a strategy to solve endo-atmospheric launch vehicle optimal control problems using indirect methods. More specifically, we combine shooting methods with an adequate continuation algorithm, taking advantage of the knowledge of an analytical solution of a simpler problem. This procedure is resumed in two main steps. We first simplify the physical dynamics to obtain a new analytical guidance law which is used as initial guess for a shooting method. Then, a continuation procedure makes the problem converge to the complete dynamics leading to the optimal solution of the original system. Numerical results are presented.

Keywords: Optimal control, Real-time control, Continuation methods, Guidance of vehicles.

1. INTRODUCTION

Guidance of autonomous launch vehicles towards rendezvous points is a complex task often considered in missile applications, mainly for interception of targets. It represents an optimal control problem, whose aim consists in finding a control law enabling the vehicle to join a final point of the 3D space considering prescribed constraints as well as performance criteria. The rendezvous point may be a static point as well as a moving point if, for example, the task consists in intercepting a maneuvering target. This requires not only *high numerical precision* of concerned algorithms but also a *real-time processing* of optimal trajectories.

The most common approach to solve this kind of task resides on *analytical guidance laws*. They correct errors coming from perturbations and misreading of the system. However, the trajectories induced by guidance laws are not optimal because of some approximations made.

Ensuring the optimality of trajectories can be achieved exploiting *direct methods*. These techniques consist in discretizing each component of the optimal control problem (the state, the control, etc.) to reduce the whole mathematical representation to a nonlinear constrained optimization problem. Since they are quite robust, they are widely used [Hargraves and Paris, 1987], [Ross et al., 2003]. However, these methods are computationally demanding and can often be used offline uniquely.

In order to manage efficiently real-time processing of optimal control sequences for launch vehicle systems one may consider *indirect methods*. These use a mathematical study of the system (exploiting the *Pontryagin Maximum Principle*) to determine some necessary conditions of optimality. Indirect methods converge much faster than direct methods with a better precision. Since the problem is equivalent to the research of zeros of a function, the main difficulty remains their *initialization*. For example, in [Pan and Lu, 2010] the initialization problem is bypassed using finite differences algorithms and multiple-shooting methods respectively. However, these approaches remain computationally demanding and not easily applicable in view of real-time processing.

In this paper, we propose to solve endo-atmospheric launch vehicle optimal control problems using indirect methods managing the issue coming from the initialization by combining an analytical guidance law and a *continuation method*. Continuation procedures have shown to be reliable and robust for problems like atmospheric reentry and coplanar orbit transfer [Cerf et al., 2012], [Trélat, 2012]. This combination allows to preserve precision and fast numerical computations. The proposed approach is resumed in two main steps. We first simplify the physical dynamics to obtain a new analytical guidance law which is used as initial guess for a shooting method. Then, a continuation procedure makes the problem converge to the complete dynamics leading to the optimal solution of the original system.

The paper is structured as follows. Section 2 introduces the dynamics of a general endo-atmospheric launch vehicle system, the optimal control formulation and the related numerical approach. Section 3 is devoted to the construction of a simplified dynamics able to initialize successfully a shooting method. In Section 4 the continuation method that recovers the original dynamics is presented with some numerical tests. Finally, Section 5 contains conclusions and perspectives.

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2. DYNAMICS, OPTIMAL CONTROL PROBLEM AND NUMERICAL METHOD

2.1 Physical Model

Let $(\mathbf{I}, \mathbf{J}, \mathbf{K})$ be an inertial frame centered at the center of the planet O, $(\mathbf{e}_L, \mathbf{e}_l, \mathbf{e}_r)$ be the NED frame and $(\mathbf{i}, \mathbf{j}, \mathbf{k})$ be the velocity frame. The endo-atmospheric launch vehicle system is modeled as an axisymmetric thrust propelled rigid body of mass m. The coordinates $(r, l, L, v, \gamma, \chi) \in \mathbb{R}^6$ (L is the latitude, l is the longitude, γ is the path angle and χ is the azimuth angle) are used to represent the position $\boldsymbol{\xi} = (r\cos(L)\cos(l), r\cos(L)\sin(l), r\sin(L))$ of the center of mass G of the vehicle and its velocity $\mathbf{v} = v\cos(\gamma)\cos(\chi)\mathbf{e}_L + v\cos(\gamma)\sin(\chi)\mathbf{e}_l - v\sin(\gamma)\mathbf{e}_r$.

Neglecting the wind velocity, the Coriolis and the centripetal force (this is legitimate because of the short length of the considered trajectories), the dynamics takes the following form

$$
\begin{aligned}
&\dot r = v\sin(\gamma)\ , \qquad \dot L = \frac{v}{r}\cos(\gamma)\cos(\chi)\ , \qquad \dot l = \frac{v}{r}\frac{\cos(\gamma)\sin(\chi)}{\cos(L)} \\
&\dot v = \frac{f_T}{m}\cos(\alpha) - (d + \eta c_m u^2)v^2 - g\sin(\gamma)\ , \qquad \dot m = -q \\
&\dot\gamma = \frac{f_T}{mv}\sin(\alpha)\cos(\beta) + v c_m u\cos(\beta) + \left(\frac{v}{r} - \frac{g}{v}\right)\cos(\gamma) \qquad (1)\\
&\dot\chi = \frac{f_T}{mv}\frac{\sin(\alpha)}{\cos(\gamma)}\sin(\beta) + \frac{v c_m}{\cos(\gamma)}u\sin(\beta) + \frac{v}{r}\cos(\gamma)\sin(\chi)\tan(L)
\end{aligned}
$$

where g is the modulus of the gravity, η is an aerodynamic efficiency factor, $\alpha = \alpha_{\max}u$ is the *angle of attack* while β is the *angle of bank*, u stands for the normalized lift coefficient while $q = q(t)$ is the mass flow and $f_T = f_T(t)$ represents the modulus of the thrust depending on $q(t)$.

Based on a standard model of flight dynamics [Pucci et al., 2015], [Pepy and Hérissé, 2014], coefficients d and c_m are approximated by $d = d(r) = \frac{1}{2m}\rho S C_{D_0}$ and $c_m = c_m(r) = \frac{1}{2m}\rho S C_{L_{\max}}$ where S is the reference area, $C_{L_{\max}}$ is the maximal value of the lift coefficient and C_{D_0} is the drag coefficient for $\alpha = 0$, which is considered constant; finally, ρ is the air density for which an exponential model $\rho = \rho(r) = \rho_0\exp(-(r - r_T)/h_r)$ is considered, where h_r is a reference altitude. Since $q(t)$ is a predefined function of time, in this paper controls are only u and β.

2.2 Optimal Control Problem and Maximum Principle

Consider now the Optimal Control Problem (**OCP**)

$$
\begin{aligned}
&\min \int_0^{t_f} f^0(t, \mathbf{x}(t), \mathbf{u}(t))\, dt \\
&\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{u}(t)) \qquad (2)\\
&\mathbf{x}(0) = \mathbf{x}_0 \in M_0\ , \quad \mathbf{x}(t_f) = \mathbf{x}_{t_f} \in M_f
\end{aligned}
$$

where $\mathbf{x}(t) = (r, L, l, v, \gamma, \chi)(t) \in \mathbb{R}^6$, $\mathbf{u}(t) = (u, \beta)(t) \in \mathbb{R}^2$, $\mathbf{f}$ is the mapping defined by dynamics (1), M_0, M_f are smooth submanifolds of $\mathbb{R}^6$ and the transfer time t_f is not fixed. Finally,

$$
f^0 = \sigma u^2 - \left(\frac{f_T}{m}\cos(\alpha) - (d + \eta c_m u^2)v^2 - g\sin(\gamma)\right) \qquad (3)
$$

where $\sigma \geq 0$ is constant. By definition, u^2 takes its values in $[0, 1]$. However, we do not consider any boundaries on u preferring to penalize it, using σ, $\eta c_m v^2$ within the cost.

The *Pontryagin Maximum Principle* (PMP) [Boltyanskiy et al., 1962] states that, if $\mathbf{u}$ is optimal with response defined on $[0, t_f]$, and shortly denoted $\mathbf{x}(t)$, then there exists $p^0 \leq 0$ and $\mathbf{p} \in AC([0, t_f], \mathbb{R}^6)$ such that

$$
\begin{aligned}
&\dot{\mathbf{x}} = \frac{\partial H}{\partial \mathbf{p}}\ , \ \dot{\mathbf{p}} = -\frac{\partial H}{\partial \mathbf{x}}\ , \ H(t, \mathbf{x}, \mathbf{p}, \mathbf{u}) = \max_{\mathbf{v}\in U} H(t, \mathbf{x}, \mathbf{p}, \mathbf{v}) \\
&\max_{\mathbf{v}\in U} H(t_f, \mathbf{x}(t_f), \mathbf{p}(t_f), \mathbf{v}) = 0 \qquad (4)
\end{aligned}
$$

a.e. on $[0, t_f]$, where $H = \langle \mathbf{p}, \mathbf{f}(t, \mathbf{x}, \mathbf{u})\rangle + p^0 f^0(t, \mathbf{x}, \mathbf{u})$ is the Hamiltonian and $\mathbf{p}$ satisfies the transversality conditions

$$
\mathbf{p}(0) \perp T_{\mathbf{x}_0}M_0\ , \quad \mathbf{p}(t_f) \perp T_{\mathbf{x}_{t_f}}M_f \qquad (5)
$$

Treating (**OCP**) by indirect methods consists in solving

$$
\begin{aligned}
&\dot{\mathbf{x}}(t) = \mathbf{f}(t, \mathbf{x}(t), \mathbf{p}(t))\ , \qquad \mathbf{x}(0) = \mathbf{x}_0\ , \ \mathbf{x}(t_f) = \mathbf{x}_f \qquad (6)\\
&\dot{\mathbf{p}}(t) = -\frac{\partial H}{\partial \mathbf{x}}(t, \mathbf{x}(t), \mathbf{p}(t))\ , \quad \mathbf{p}(0) = \mathbf{p}_0
\end{aligned}
$$

with an appropriate value for $\mathbf{p}_0$.

2.3 Shooting and Continuation Method

It is known [Trélat, 2008] that finding a solution of (6) can be reduced to solve $G(\mathbf{p}_0, t_f) = 0$, $G : \mathbb{R}^{n+1} \longrightarrow \mathbb{R}^{n+1}$ (G is called shooting function) using Newton-type methods. This is the content of the well known *shooting method* in optimal control [Trélat, 2008], [Stoer and Bulirsch, 2013]. Its advantage is its extremely good numerical accuracy, relevant for aerospace applications [Trélat, 2012]. Since it relies on the Newton method, it inherits of the very quick convergence properties of the Newton method. Its main drawback is that it may be difficult to initialize.

To overcome this difficulty, one can entrust with the robustness of the *continuation method*. It consists in deforming the problem into a simpler one that we are able to solve and then in solving a series of shooting problems, step by step by parameter deformation, to recover the original problem [Allgower and Georg, 2003]. This approach increases the efficiency of the shooting method because it allows to relax its initialization. The continuation parameter λ may be a physical parameter (or several) of the problem, or an artificial one. The path consists of a convex combination of the simpler problem and of the original one.

The main algorithm proposed consists then in finding a solution of some simplification of (**OCP**) first and, from this, solving by continuation the original formulation (**OCP**).

3. NEW GUIDANCE LAW AS A GOOD ESTIMATE FOR THE CONTINUATION METHOD

Continuation methods allow us to solve iteratively (**OCP**) once the solution of some (usually) simpler optimal control problem is known. Here, we introduce and treat an efficient simpler problem coming from a modification of (1).

This modified version of (**OCP**) is designed with the hope that, on one hand, the shooting method can be easily