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Implicit Integrators for Linear Dynamics Coupled to a Nonlinear Static Feedback and Application to Wind Turbine Control *

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Abstract: Efficient integration schemes with sensitivity propagation are crucial for deploying real-time Nonlinear Model Predictive Control on systems described by continuous time dynamics. Implicit integration schemes are preferred when stiff modes are present in the model equations, or when the equations are implicit. We consider here a class of models, where the dynamics are linear, but coupled to a general static nonlinear feedback function. We propose a collocation-based implicit integration scheme where a lifting-condensing approach is used to exploit this specific structure to reduce the size of the linear algebra underlying the integrator. This technique yields a significant reduction in the computational complexity of performing the system integration and sensitivity analysis, when the static nonlinearity is of much smaller dimensions than the complete dynamics. The proposed method is illustrated on a complex wind turbine model, resulting in a significant gain of computational time in the linear algebra, and an overall gain of computational time of a factor 2.

Keywords: Real-time NMPC, implicit integrators, structure-exploitation, wind turbine

1. INTRODUCTION

Nonlinear Model Predictive Control (NMPC) (Mayne and Rawlings, 2013) and Moving Horizon Estimation (MHE) (Rao, 2000) are successful approaches to tackle real-time optimal control and estimation, because they can explicitly handle constraints and nonlinear dynamics. For systems having fast dynamics, the high computational demand of NMPC and MHE is a major challenge for their real-time deployment. Indeed, an Optimal Control Problem (OCP) needs to be solved at every sampling time, while respecting the time constraints imposed by the real-time application. Recent algorithmic progress (Diehl et al., 2009; Kirches et al., 2010) allows to consider NMPC and MHE for fast systems. Among the available online algorithms, the Real-Time Iteration (RTI) scheme (Diehl et al., 2002) is a very successful approach.

A central component in deploying NMPC and MHE schemes is the handling of the nonlinear continuous time dynamics that represent the physical evolution of the real system. NMPC and MHE require numerical simulations of these dynamics, together with the propagation of the sensitivities of these simulations for direct optimal control (Bock and Plitt, 1984). Many of these simulations have to be carried out at every sampling time of the NMPC or MHE scheme, for each OCP solution. This often forms one of the computational bottlenecks of deploying NMPC and MHE in real time.

For systems having stiff or implicit dynamics, implicit integration schemes are typically preferred as they allow for performing the simulations of the system model at a lower computational cost than explicit schemes (Hairer and Wanner, 1991). Implicit integrators require, in most cases, solving numerically a set of nonlinear equations for each integration step. This nonlinear system can be solved via a Newton iteration, which requires the deployment of an efficient, typically dense linear algebra within the integrator. The corresponding matrix factorizations are then typically the computational bottleneck of the numerical integration scheme (Quirynen et al., 2012).

For small to medium-scale systems, which are relatively dense, one typically uses dense linear algebra routines. The computational complexity of the matrix factorizations grows cubically with the problem size (Golub and Loan, 1996). Recent research work has been proposed to exploit structures commonly present in dynamic models in order to reduce the size of the linear algebra underlying the integrator and therefore its computational cost, e.g., see (Quirynen et al., 2013). An alternative is to use

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a structured Jacobian approximation in a Newton-type implementation as proposed in (Butcher, 1976).

In this paper, we are interested in model structures where a large linear dynamic system is coupled to a smaller, static nonlinear feedback. More specifically, we consider dynamic models having the following structure:

$$E\dot{x} = Ax + Bu + C\phi\left(Dx + F\dot{x}, u\right), \tag{1}$$

where $x \in \mathbb{R}^{n_x}$ and $\phi : \mathbb{R}^{n_{\text{in}}} \mapsto \mathbb{R}^{n_{\text{out}}}$. For the sake of simplicity, we will consider that matrices $A, E \in \mathbb{R}^{n_x \times n_x}$, $B \in \mathbb{R}^{n_x \times n_u}$, $D, F \in \mathbb{R}^{n_{\text{in}} \times n_x}$ and $C \in \mathbb{R}^{n_x \times n_{\text{out}}}$ are constant. We will be interested in systems where $n_{\text{out}} \ll n_x$, i.e., where the linear dynamics have a significantly larger dimension than the nonlinear feedback function.

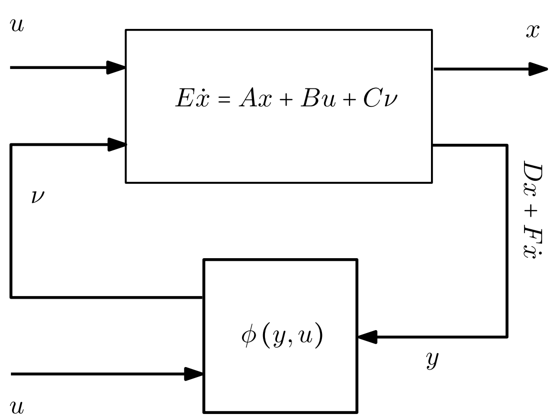

Fig. 1. Schematic of the dynamics considered in this paper.

Figure 1 illustrates the dynamic model structure. It arises in many mechanical applications where, e.g., structural vibrations are modeled and yield large linear dynamics, while being driven by a relatively smaller number of forces that depend nonlinearly on the inputs, states, and possibly on their time derivatives. We show in this paper that the linear dynamics and the nonlinear static feedback in the dynamic system can be exploited in collocation based implicit integrators (Quirynen et al., 2016a). This can significantly reduce the size of the linear system solutions underlying the integrator in the proposed implementation.

The paper is organized as follows. Section 2 provides preliminaries for the paper. Section 3 presents the proposed method for reducing the complexity of the linear algebra underlying the collocation-based integrator. Section 4 presents an application example of a complex, industrial wind turbine model. Section 5 presents the conclusions.

Contribution: this paper presents a method to perform the numerical integration of a specific class of nonlinear dynamic models via collocation-based integration schemes, which can allow for a large reduction of the computational complexity of the numerical integration.

2. DIRECT OPTIMAL CONTROL

We consider NMPC schemes, which solve an optimal control problem in the form:

$$\begin{aligned}
\min_{x(\cdot),u(\cdot)} \quad & \|x(t+T) - \bar{x}\|_P^2 + \\
& \int_t^{t+T} \|x(\tau) - \bar{x}\|_Q^2 + \|u(\tau) - \bar{u}\|_R^2 \,\mathrm{d}\tau && \text{(2a)}\\
\text{s.t.} \quad & x(t) - \hat{x}(t) = 0 && \text{(2b)}\\
& F\left(\dot{x}(\tau), x(\tau), u(\tau)\right) = 0, \quad \forall \tau \in [t, t+T] && \text{(2c)}\\
& h\left(x(\tau), u(\tau)\right) \le 0, && \text{(2d)}
\end{aligned}$$

where $x(\cdot) \in \mathbb{R}^{n_x}$ and $u(\cdot) \in \mathbb{R}^{n_u}$ are the states and inputs, respectively, and $\bar{x}(\cdot)$ and $\bar{u}(\cdot)$ are the corresponding reference trajectories. The value T denotes the NMPC prediction horizon and $\hat{x}(t)$ is the state estimation at the current time t. The dynamics of the physical system are captured in the model function F, where the differential state derivatives $\dot{x}(\cdot)$ are defined implicitly. Function h gathers the constraints imposed on the control problem. Note that MHE requires the solution of an OCP with a very similar structure (Diehl et al., 2009).

2.1 Direct Multiple Shooting

A successful approach for treating problem (2) numerically is the Multiple Shooting method (Bock and Plitt, 1984), where the control input is typically parameterized as a piecewise-constant over a uniform time grid

$$u(\tau) = u_k \quad \text{for} \quad \tau \in [t_k, t_{k+1}), \tag{3}$$

for $k = 0, \ldots, N-1$ and where $t_N - t_0 = T$. The states can be discretized on the same time grid, taking the values $x_0, \ldots, x_N$. The continuous time dynamics are handled separately on each time interval $[t_k, t_{k+1}]$ via numerical integration. Let us define the function $f(x_k, u_k)$, which for a state value x_k and a piecewise constant input u_k at time t_k, delivers a simulation of the dynamics (2c) over the time interval $[t_k, t_{k+1}]$. The OCP in Eq. (2) is then typically parameterized as:

$$\begin{aligned}
\min_{x,u} \quad & \|x_N - \bar{x}\|_P^2 + \sum_{k=0}^{N-1} \|x_k - \bar{x}\|_Q^2 + \|u_k - \bar{u}\|_R^2 && \text{(4a)}\\
\text{s.t.} \quad & x_0 - \hat{x}(t) = 0 && \text{(4b)}\\
& f(x_k, u_k) - x_{k+1} = 0, \quad k = 0, \ldots, N-1 && \text{(4c)}\\
& h(x_k, u_k) \le 0. && \text{(4d)}
\end{aligned}$$

Our focus here is to propose an efficient evaluation of the numerical integration in (4c) for the specific dynamic structure in (1). As discussed also further, it is additionally important to be able to efficiently evaluate first and possibly higher order derivatives (Griewank, 2000; Quirynen et al., 2016b) when using Newton-type optimization (Nocedal and Wright, 2006) to solve the nonlinear optimization problem in (4).

2.2 Collocation-based Integration

A collocation-based integration of the dynamics in (1) is based on a polynomial approximation of the state trajectories $x(t)$ on the shooting intervals $[t_k, t_{k+1}]$:

$$x(\mathbf{s}_k, t) = \sum_{i=0}^{K} s_{k,i} P_{k,i}(t), \quad \text{for} \quad t \in [t_k, t_{k+1}], \tag{5}$$

where $s_{k,i}$ are the collocation variables, which we will note:

$$\mathbf{s}_k = \begin{bmatrix} s_{k,0} \\ \ldots \\ s_{k,K} \end{bmatrix} \tag{6}$$