

## Nonlinear Model Predictive Congestion Control for Networks

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**Abstract:** This paper addresses congestion control for networks. The main focus is the modeling and compensation of nonlinear disturbance in the network, which is seldom considered in the literature. A new congestion control method is presented to compensate for the effects of nonlinear disturbance, uncertainty, time-varying delay, and input constraint. A state feedback congestion controller is designed via model predictive control approach. The stability of the closed-loop system is analyzed by using Lyapunov-Krasovskii functional. The effectiveness and feasibility of the proposed controller are verified by simulation results.

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**Keywords:** Congestion control, model predictive control, nonlinear disturbance, uncertainty, time-varying delay, input constraint.

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### 1. INTRODUCTION

Congestion is a serious problem for networks (Jacobson, 1988; Low et al., 2002), which will reduce the quality of service (QoS). In order to avoid congestion in transmission control protocol (TCP), active queue management (AQM) is an effective congestion control approach, and a lot of AQM-based congestion control methods have been reported in the literature, such as random early detection (RED) (Floyd and Jacobson, 1993; Chen and Yang, 2009), proportional-integral (PI) (Hollot et al., 2002), proportional-integral-derivative (PID) (Fan et al., 2003), coefficient diagram method (CDM) (Bigdeli and Haeri, 2009a), adaptive technique (Barzamini et al., 2012; Patete et al., 2008; Zhang et al., 2003), robust control (Chen and Yang, 2007; Quet and Özbay, 2004; Tan et al., 2007; Zheng and Nelson, 2009), and sliding mode control (Ignaciuk and Bartoszewicz, 2011, 2012). However, all the proposed controllers did not consider constraint nonlinearity in input. In fact, the drop probability considered as an input is limited between 0 and 1 in real TCP/IP networks. Therefore, the effect of a saturating input should be considered.

In order to deal with the input constraint, Chen et al. (2007) presented a robust congestion controller for TCP/AQM system, but did not consider the uncertainty and disturbance; moreover, they only considered the constant time delay in state, while did not consider the time delay in control input. Additionally, the congestion controllers mentioned above are designed for continuous time systems. Sall et al. (2011) presented a robust digital congestion control algorithm, but did not consider the uncertainty and disturbance.

It is well known that the model predictive control (MPC) has strong ability to cope with time delay, uncertainty, and input constraint (Kothare et al., 1996; Mayne et al., 2000, Yan and Bitmead, 2005). A congestion controller based on MPC was presented in Wang et al. (2012). An implementation method of MPC as a digital controller was investigated in Marami and Haeri (2010). A predictive functional control (PFC) was introduced as a new AQM controller in Bigdeli and Haeri (2009b). However, the nonlinear disturbance, uncertainty, input constraint, and time-varying state and control input delays are not considered in these predictive controllers.

In the previous work (Han et al., 2014), we presented a model predictive congestion control for networks with uncertainty, input constraint, and time-varying delays in both state and control input, but did not consider the nonlinear disturbance. In fact, from the dynamics of the AQM (see details in the next section), the nonlinear disturbance should be considered in the design stage of the controller. To the best knowledge of the authors, the nonlinear disturbance in congestion control system is seldom considered in the literature.

This paper presents a model predictive congestion control method for networks with nonlinear disturbance, uncertainty, input constraint, and time-varying state and input delays. The state feedback congestion controller is designed by using linear matrix inequality (LMI). Simulation results show that the proposed method has good performance. In addition, the discrete-time congestion control model is used to obtain a digital MPC controller directly.

This paper is organized as follows: The problem formulation is described in Section 2. A model predictive congestion controller is designed in Section 3. Simulation results are displayed in Section 4. And conclusion remarks are addressed in Section 5.

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## 2. PROBLEM FORMULATION

Consider the following congestion control system (Zheng and Nelson, 2009)

$$\begin{cases} \dot{w}(t) = \frac{1}{\tau(t)} - \frac{w(t)}{2} \frac{w(t-\tau_0)}{\tau(t-\tau_0)} p(t-\tau_0), \\ \dot{q}(t) = \begin{cases} -c(t) + \frac{N(t)}{\tau(t)} w(t), & q(t) > 0, \\ \max \left\{ 0, -c(t) + \frac{N(t)}{\tau(t)} w(t) \right\}, & q(t) = 0, \end{cases} \\ \tau(t) = \frac{q(t)}{c(t)} + T_p, \end{cases} \quad (1)$$

where  $w(t)$  is the TCP window size (packets),  $\tau(t)$  the round-trip time delay (secs),  $p(t)$  the probability of packet mark,  $q(t)$  the queue length (packets),  $c(t)$  the available link capacity (packets/s),  $T_p$  propagation delay (secs), and  $N(t)$  the number of sessions.

The operating point is:  $w_0 = (\tau_0 c) / N$ ,  $p_0 = 2 / w_0^2$ ,  $\tau_0 = (q_0 / c) + T_p$ , which can be obtained from  $\dot{w}(t) = 0$  and  $\dot{q}(t) = 0$ . Through the linearization around  $(w_0, q_0, p_0)$ , (1) can be written by (Zheng and Nelson, 2009)

$$\begin{cases} \delta \dot{w}(t) = -\frac{N}{\tau_0^2 c} [\delta w(t) + \delta w(t-\tau_0)] \\ \quad - \frac{1}{\tau_0^2 c} [\delta q(t) - \delta q(t-\tau_0)] \\ \quad - \frac{\tau_0 c^2}{2N^2} \delta p(t-\tau_0) \\ \quad + \frac{\tau_0 - T_p}{\tau_0^2 c} [\delta c(t) - \delta c(t-\tau_0)] \\ \delta \dot{q}(t) = \frac{N}{\tau_0} \delta w(t) - \frac{1}{\tau_0} \delta q(t) - \frac{T_p}{\tau_0} \delta c(t) \end{cases}, \quad (2)$$

where  $\delta w = w - w_0$ ,  $\delta q = q - q_0$ ,  $\delta p = p - p_0$ ,  $\delta c = c - c_0$ .

The state space form of (2) becomes

$$\dot{x}(t) = A_s x(t) + A_{ds} x(t-\tau) + B_s u(t-\tau) + D_s v(t), \quad (3)$$

where  $x(t) = [\delta w(t), \delta q(t)]^T$  is the state,  $u(t) = \delta p(t)$  is the control input,  $v(t) = [\delta c(t), \delta c(t-\tau)]^T$  is the disturbance, and

$$A_s = \begin{bmatrix} -\frac{N}{\tau_0^2 c} & -\frac{1}{\tau_0^2 c} \\ \frac{N}{\tau_0} & -\frac{1}{\tau_0} \end{bmatrix}, \quad A_{ds} = \begin{bmatrix} -\frac{N}{\tau_0^2 c} & -\frac{1}{\tau_0^2 c} \\ 0 & 0 \end{bmatrix},$$

$$B_s = \begin{bmatrix} -\frac{\tau_0 c^2}{2N^2} \\ 0 \end{bmatrix}, \quad D_s = \begin{bmatrix} \frac{\tau_0 - T_p}{\tau_0^2 c} & -\frac{\tau_0 - T_p}{\tau_0^2 c} \\ -\frac{T_p}{\tau_0} & 0 \end{bmatrix}.$$

By discretizing (3) (Astrom and Wittenmark, 1997), and noting that the disturbance  $v(t) = [\delta c(t), \delta c(t-\tau)]^T$  is a nonlinear function since  $c(t)$  in  $\tau(t) = q(t) / c(t) + T_p$  is nonlinear, the discrete-time model is consequently given by

$$x(k+1) = Ax(k) + A_d x(k-\tau(k)) + Bu(k-\tau(k)) + f(k, x(k)), \quad (4)$$

where  $\tau(k)$  is time-varying delay,  $f(k, x(k))$  is the nonlinear disturbance, and  $A = e^{A_s T_s}$ ,  $A_d = e^{A_{ds} T_s}$ ,  $B = \int_0^{T_s} e^{A_s t} B_s dt$ .

Considering the unmodeled uncertainty, we can rewrite (4) as  $x(k+1) = A(k)x(k) + A_d(k)x(k-\tau(k)) + B(k)u(k-\tau(k)) + f(k, x(k))$ , (5)

where  $A(k) = A + \Delta A(k)$ ,  $A_d(k) = A_d + \Delta A_d(k)$ ,  $B(k) = B + \Delta B(k)$ , and  $\Delta A(k)$ ,  $\Delta A_d(k)$  and  $\Delta B(k)$  are the time-varying uncertain matrices of the form (Chen and Yang, 2007)

$$[\Delta A(k), \Delta A_d(k), \Delta B(k)] = DF(k)[E_1, E_2, E_3], \quad (6)$$

with known constant matrices  $D$ ,  $E_i$ , and  $F(k)^T F(k) \leq I$ .

Suppose that the time-varying delay  $\tau(k)$  satisfy

$$0 \leq \tau_m \leq \tau(k) \leq \tau_M, \quad (7)$$

where  $\tau_m$  and  $\tau_M$  are known the lower and upper bounds of time delay. The constraint of control input is given by

$$-\bar{u} \leq u(k+i) \leq \bar{u}, \quad i \geq 0. \quad (8)$$

And from  $c(t) = q(t) / [\tau(t) - T_p]$ , the nonlinear disturbance  $f(k, x(k))$  is state-dependent, then it is supposed that

$$\|f(k, x(k))\| \leq \beta_1 \|x(k)\| + \beta_2 \|x(k-\tau_k)\|, \quad (9)$$

where  $\beta_1$  and  $\beta_2$  are known constants.

Define

$$\min_{u(k+j|k), j \geq 0} \max_{[\Delta A(k+j), \Delta A_d(k+j), \Delta B(k+j)] \in \Omega} J(k), \quad (10)$$

subject to

$$J(k) = \sum_{j=0}^{\infty} \{ \|x(k+j|k)\|_{Q_1}^2 + \|u(k+j|k)\|_R^2 \}, \quad (11)$$

$$\begin{aligned} x(k+j+1|k) &= A(k+j)x(k+j|k) + A_d(k+j) \\ &\quad \times x(k+j-\tau_k|k) + Bu(k+j|k) + f(k+j), \end{aligned} \quad (12)$$

$$-\bar{u} \leq u(k+j) (= Kx(k+j|k)) \leq \bar{u}, \quad j \in [0, \infty), \quad (13)$$

where  $J(k)$  is the cost function,  $Q_1 > 0$  and  $R > 0$  are weighting matrices,  $x(k+j|k)$  is the value of  $x$  at future time  $k+j$  ( $j > 0$ ) predicted at the time  $k$ ,  $\Omega$  is the set of uncertainties, i.e.,  $[\Delta A(k), \Delta A_d(k), \Delta B(k)] \in \Omega$ .

The problem of this paper is to design a state feedback controller  $u(k) = Kx(k)$  for system (5) with parameter uncertainty (6), time-varying delay (7), input constraint (8), and nonlinear disturbance (9), by solving the optimization problem (10)-(13).

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