Available online at www.sciencedirect.com

ScienceDirect

IFAC PapersOnLine 50-1 (2017) 588–593

Control design for thermodynamic systems on contact manifolds

Nicolas Hudon [*] **Martin Guay** [*] **Denis Dochain** [**]

[*] *Department of Chemical Engineering, Queen's University, Kingston, ON, Canada* (nicolas.hudon@queensu.ca, martin.guay@chee.queensu.ca)

[**] *ICTEAM, Université catholique de Louvain, Louvain-la-Neuve, Belgium* (denis.dochain@uclouvain.be)

Abstract: We consider the problem of feedback control design for thermodynamic systems. Following past contributions, we consider the expression of closed-loop control in the Thermodynamic Phase Space (TPS) using contact geometry. Some of these past contributions identified restricted classes of admissible controllers by restricting the dynamics to a Legendre submanifold in the TPS. In this paper, we consider the problem of controlling a system through damping feedback control. By allowing this particular class of feedback design technique, we characterize the impact of control on the thermodynamic structure of the closed-loop system in the TPS. An example is considered to illustrate the proposed construction.

Keywords: Thermodynamic, Nonlinear damping feedback, Feedback control design, Contact geometry

1. INTRODUCTION

An approach to study thermodynamic systems is to use contact geometry, as an analogue of symplectic geometry for classical mechanics (Arnold, 1989). This approach, developed extensively to describe equilibrium thermodynamics (Mrugala et al., 1991). In the context of control systems analysis and feedback design for thermodynamic systems, contact geometry was considered, through a lift of a known control system, in (Eberard et al., 2007), (Favache et al., 2009), (Favache et al., 2010), (Gromov and Caines, 2015), (Ramirez et al., 2013b), and more recently in (Wang et al., 2015). Stability analysis problems were successfully addressed for control systems using the contact geometry approach, see for example the contribution by Maschke (2016). As presented in (Favache et al., 2010), both the energy and entropy functions can serve as the generating potential for the representation of thermodynamic systems using contact geometry. The Thermodynamic Phase Space (TPS) approach has the advantage of encoding both laws of thermodynamics in the expression of a system dynamics in an extended phase space (Grmela, 2002). The construction proposed in (Grmela, 2002), built on material from (Arnold, 1989), shows that the thermodynamic reciprocity relations are encoded within this framework. Contact geometry also serves as the basis for the geometrothermodynamics approach to nonequilibrium thermodynamics, see for example the original contribution (Quevedo, 2007) and applications presented in (Quevedo et al., 2011) and (Quevedo and Tapias, 2014), where the TPS is endowed with a metric, in the spirit of Weinhold and Ruppeiner (Quevedo, 2007), *i.e.*, by using the Hessian of the thermodynamic potential as a metric. An indefinite Riemannian metric was also introduced on the TPS in (Mrugala, 1996), a construction later used in (Preston and Vargo, 2008) to study geometric properties of constitutive surfaces defined for different thermodynamic potentials.

The results developed in (Eberard et al., 2007; Favache et al., 2009, 2010; Ramirez et al., 2013b; Wang et al., 2015; Gromov and Caines, 2015) were key to understand stability and stabilization problems for thermodynamic systems: By lifting a n-dimensional controlled dynamics to a $(2n + 1)$-dimensional dynamical systems endowed with a contact structure, *i.e.*, a differential one-form encoding thermodynamics evolution constraints, it is possible to restrict stability and stabilization problems to admissible evolutions in an extended vector field. A further contribution to this understanding about the interplay between thermodynamic and feedback control is given in (Ramirez et al., 2013a) where two results pertinent to the present study were presented. First, it was demonstrated that the only state feedback preserving a given contact structure of a conservative control contact system is the constant one. Second, a feedback design approach as solution to matching equations, as used for passivity-based damping assignment for port-controlled Hamiltonian (Astolfi and Ortega, 2009), was considered in the TPS. The feedback design approaches from the contribution (Ramirez et al., 2013a) were characterized as restrictive, see for example the review given in (Maschke, 2016) where a characterization of the desired properties of controlled system in the TPS was proposed. However, it is not clear how far from the working assumptions from (Ramirez et al., 2013a) it is possible to go such that the resulting closed-loop system preserves the thermodynamic structure of the original system. The contribution(Maschke, 2016) pointed out the importance of the lift to the TPS as a key factor to assess stability (and indirectly feedback stabilization), as the choice of a lift is linked to the definition of the Reeb

Peer review under responsibility of International Federation of Automatic Control.
10.1016/j.ifacol.2017.08.084

vector field, which is central to the analysis in the TPS provided in (Ramirez et al., 2013a).

To clarify our understanding of the relations between control design and the thermodynamic representation approach based on contact geometry, we consider a particular class of physics-based feedback control design *prior* to the performing a lift of the dynamics to the TPS. Following previous investigations on damping feedback control (Hudon and Guay, 2013), we study systems in closed-loop in the TPS for which the control action has some physical sense, as for example damping feedback for port-controlled Hamiltonian systems (Ortega et al., 2002). With this study, we hope to shed a light on the admissible class of controllers, *i.e.*, feedback controllers preserving the thermodynamic structure of a given open-loop system.

This paper is organized as follows. Necessary background on feedback control in the TPS is given in Section 2. In Section 3, the lift of systems in closed-loop with damping feedback controllers is studied. An example is given in Section 4. Conclusions and future areas for investigation are discussed in Section 5.

2. BACKGROUND

In this section, we summarize the theory of thermodynamic systems expressed in the TPS, that is, using contact geometry, following the exposition proposed originally in (Eberard et al., 2007) and developed in (Ramirez et al., 2013a; Maschke, 2016) for control design and feedback stabilization.

We denote the n extensive variables by x^i, $i = 1, \ldots, n$, and the thermodynamic potential by x^0, for example the energy $x^0 = E(x)$ or the Entropy $x^0 = S(x)$. The n intensive variables are denoted by p_i and are dual to the extensive variables by the relations $p_i = \frac{\partial E}{\partial x^i}$ or $p_i = \frac{\partial S}{\partial x^i}$, depending on the choice of thermodynamic potential[1]. The thermodynamic phase space (TPS) is the $(2n+1)$-dimensional vector space endowed with the canonical contact structure

$$\theta = dx^0 + \sum_{i=1}^{n} p_i dx^i.$$

Definition 1. A one-form θ on a $2n+1$-dimensional manifold $\mathcal{M}$ is a contact form if $\theta \wedge (d\theta)^n \neq 0$ is a volume form. Then the pair $(\mathcal{M}, \theta)$ is called a contact manifold.

For a given set of canonical coordinates and any partition I and J of the set of indices $\{1, \ldots, n\}$, for any differentiable function $\phi(x^I, p_J)$ of n variables, $i \in I$, $j \in J$, the formulas

$$\begin{aligned} x^0 &= \phi - \sum_{i \in I} p_i \frac{\partial \phi}{\partial p_i} \\ x^i &= -\frac{\partial \phi}{\partial p_i}, i \in I, \\ p_j &= \frac{\partial \phi}{\partial x^j}, j \in J, \end{aligned} \tag{1}$$

define a Legendre submanifold $\mathcal{L}_\phi$ of $\mathbb{R}^{2n+1}$.

An important object to be studied in our context is the vector field governing the dynamics of a system in the TPS.

Definition 2. A (smooth) vector field X on the contact manifold $\mathcal{M}$ is a contact vector field with respect to a contact form θ if and only if there exists a smooth function $\rho \in C^\infty(\mathcal{M})$ such that

$$L_X \theta = \rho \theta,$$

where $L_X\cdot$ denotes the Lie derivative with respect to the vector field X. It is called a strict contact vector field if $\rho = 0$.

The problem considered in (Ramirez et al., 2013a; Maschke, 2016) is to study controlled balance equation system of the form

$$\frac{dx}{dt} = f(x, \phi_x) + \sum_{j=1}^{p} g^j(x, \phi_x) u_j, \tag{2}$$

where the gradient of the thermodynamic potential $\phi(x)$, $\frac{\partial \phi}{\partial x}$, is denoted by ϕ_x. The approach proposed in (Ramirez et al., 2013a; Maschke, 2016) first considers the lift of the controlled balance system (2) to the complete Thermodynamic Phase Space by defining the following contact Hamiltonian functions:

$$K_0 = (\phi_x - p)^T f(x, \phi_x) \tag{3}$$

$$K_C^j = (\phi_x - p)^T g^j(x, \phi_x). \tag{4}$$

By construction, the contact vector field X_K, generated by the function $K = K_0 + \sum_{j=1}^{p} K_C^j$, leaves invariant the Legendre submanifold $\mathcal{L}_\phi$ generated by the thermodynamic potential $\phi(x)$, defined as

$$\mathcal{L}_\phi = \{x_0 = \phi(x), x = x, p = \phi_x, x \in \mathbb{R}^n\}. \tag{5}$$

To every function K, there corresponds the contact vector field $\mathcal{X}_K$ given as

$$\begin{aligned} \mathcal{X}_K = &\left(K - \sum_{i=1}^{n} p_i \frac{\partial K}{\partial p_i}\right) \frac{\partial}{\partial x^0} + \frac{\partial K}{\partial x^0} \left(\sum_{i=1}^{n} p_i \frac{\partial}{\partial p_i}\right) \\ &+ \sum_{j=1}^{n} \left(\frac{\partial K}{\partial x^j}\frac{\partial}{\partial p_j} - \frac{\partial K}{\partial p_j}\frac{\partial}{\partial x^j}\right). \end{aligned} \tag{6}$$

The corresponding dynamical system in the contact phase space is given as

[1] Generally speaking, any thermodynamic potential could be used, internal energy, entropy, Helmholtz free energy, or the Gibbs free energy. Those representations are related by Legendre transformations (?). The proper choice of a potential depends on the particular problem at hand. We do not make a particular choice here and in the sequel, and the thermodynamic potential is denoted by $\phi(x)$.