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Controlling a triangular flexible formation of autonomous agents. ⋆

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Abstract: In formation control, triangular formations consisting of three autonomous agents serve as a class of benchmarks that can be used to test and compare the performances of different controllers. We present an algorithm that combines the advantages of both position- and distance-based gradient descent control laws. For example, only two pairs of neighboring agents need to be controlled, agents can work in their own local frame of coordinates and the orientation of the formation with respect to a global frame of coordinates is not prescribed. We first present a novel technique based on adding artificial biases to neighboring agents' range sensors such that their eventual positions correspond to a collinear configuration. Right after, a small modification in the bias terms by introducing a prescribed rotation matrix will allow the control of the bearing of the neighboring agents.

Keywords: Formation control, Distributed control, Multi-agent system.

1. INTRODUCTION

The theory of rigidity and in particular the concepts of *infinitesimal and minimal rigidity* have been proven to be very useful in formation control with the goal to define and achieve prescribed shapes by just controlling the distances between neighboring agents (see Anderson et al. (2008) and Krick et al. (2009)). The popular distance-based gradient descent algorithm for rigid formations has appealing properties. For example, the agents can work in their own local frame of coordinates, the system can be made robust against biased sensors, and since the orientation of the desired shape is not prescribed, one can induce rotational motions to the formation as presented in Garcia de Marina et al. (2015, 2016). In comparison, position-based formation control needs a common frame of coordinates and the steady-state orientation of the shape is restricted, which implies that a free rotational motion is not allowed; however, the same formation shape can be achieved by fewer pairs of neighboring agents in position-based than in distance-based control. This prompts a search for compromise among the advantages of both formation control techniques.

The minimum total number of pairs of neighboring agents in a distance-based rigid formation control system is given by the necessary conditions for infinitesimal and minimal rigidity, which is equal to $2n-3$ for 2D scenarios, where n is the total number of agents. This is directly related to the necessary number of inter-agent distances to be controlled such that the desired shape is (at least locally) uniquely defined. For example, the triangle is the simplest rigid shape in the 2D case and the necessary number of desired distances to define such a shape is three. On the other hand, one can achieve a desired triangular shape by just controlling (the relative positions rather than distances of) two neighboring agent pairs in the position-based setup. We illustrate these basic concepts in Figure 1.

This paper focuses on triangular formations consisting of three agents. This apparently simple setup has been considered as a benchmark in formation control (see Cao et al. (2007); Anderson et al. (2007); Cao et al. (2008); Liu et al. (2014); Mou et al. (2014)), since it allows detailed rigorous analysis for novel techniques as we aim at in this work. This provides a starting point in order to achieve more general formations. The goal of this paper is to propose an algorithm that combines the advantages of both distance- and position-based control, i.e., agents employ their own local frames of coordinates, no prescribed orientation for the desired shape (so rotational motions are allowed) and

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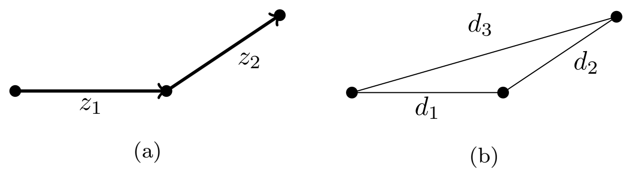

Fig. 1. The same triangular shape depicted by the three agents (dots) can be achieved by controlling the two vectors z_1 and z_2 (position-based control) or the three distances d_1, d_2 and d_3 (distance-based control). Note that in a) the orientation of the triangle is prescribed while it is not in b).

smaller number of controlled pairs of neighboring agents than in distance-based minimally rigid formations.

The novelty of this algorithm lies in exploiting the sensing-error-induced collective motion, e.g., those caused by biased range sensors among neighboring agents. For example, it has been reported in Mou et al. (2016) that small constant biases in range sensors for a rigid formation cause the whole team of agents to converge to a distorted version of the desired shape and eventually it will exhibit some steady-state motion. On the other hand, it has been shown in Garcia de Marina et al. (2016) that one can introduce artificially such biases in order to steer the desired rigid formation in a controlled way.

This paper will first study the effect of these biases in a *flexible* (or non-rigid) distance-based formation of three agents, i.e., we control only two distances instead of three for a rigid triangular formation. We prove that if the range measurements between agents are not perfectly accurate then the three agents converge to collinear positions. This is somewhat counter-intuitive since for a non-rigid formation the steady-state shape is, depending on the initial conditions, arbitrary within a constraint set. Furthermore, we will show that depending on the value of these biases, the eventual collinear formation will move with a constant velocity or remain stationary. In our first step, we take advantage of this effect in order to align the three agents with the desired distances between them. Then, we will show how a small modification in this *biased* control law can achieve the control of the angle between the two relative vectors of the flexible formation, leading to the control of a stationary desired triangular shape with no restriction on its orientation.

In the following section we introduce some notations, briefly review the (exponential) stability of formation systems under the *flexible* shape and introduce the biases in the range measurements. We continue in Section 3 analyzing the consequences of such biases in the control of a non-rigid formation. We introduce a technique motivated by Mou et al. (2016), where the authors study non-minimally but infinitesimally rigid formations by *reducing* the problem to minimally rigid ones. In this paper since we are dealing with fewer pairs of neighboring agents than in minimally rigid setups, we instead *augment* our formation to a minimally rigid one in order to study the stability of a biased non-rigid formation. In Section 4 we introduce and analyze a modified version of the *biased* algorithm in order to control triangular shapes. We finish the paper with simulations in Section 5 and some conclusions in Section 6.

2. NON-RIGID FORMATION OF THREE AGENTS

2.1 Gradient descent distance-based formation control

We consider a team of three agents governed by the first-order kinematic model

$$\dot{p}_i = u_i, \tag{1}$$

where $p_i \in \mathbb{R}^2$ is the position of the agent $i = \{1, 2, 3\}$, and $u_i \in \mathbb{R}^2$ is the control action over agent i.

Let us define the two vectors

$$z_1 \stackrel{\Delta}{=} p_1 - p_2, \quad z_2 \stackrel{\Delta}{=} p_2 - p_3, \tag{2}$$

which are the relative positions corresponding to the two links available to the agents. For each link one can construct a potential function $V_k, k \in \{1, 2\}$ with its minimum at the desired distance d_k, so that the gradient of such functions can be used to control inter-agent distances distributively. We consider the following *shape potential function*

$$V_k(z_k) = \frac{1}{2}(||z_k|| - d_k)^2, \tag{3}$$

with the following gradient along z_k

$$\nabla_{z_k} V_k(z_k) = \hat{z}_k(||z_k|| - d_k), \tag{4}$$

where $\hat{z}_k \stackrel{\Delta}{=} \frac{z_k}{||z_k||}$. We then apply to each agent i in (1) the following gradient descent control

$$u_i = -\nabla_{p_i} \sum_{k=1}^{2} V_k(z_k), \tag{5}$$

and by denoting the distance error for the kth link by

$$e_k = ||z_k|| - d_k, \tag{6}$$

we arrive at the following dynamics

$$\begin{cases} \dot{p}_1 &= -\hat{z}_1 e_1 \\ \dot{p}_2 &= \hat{z}_1 e_1 - \hat{z}_2 e_2 \\ \dot{p}_3 &= \hat{z}_2 e_2. \end{cases} \tag{7}$$

System (7) has some interesting properties. For example, the agents can employ their own local systems of coordinates, i.e., a global or common frame of coordinates is not necessary, and collision avoidance is guaranteed among pairs of neighboring agents (Oh et al. (2015)). For *tree graph* topology, one can prove the (almost global) exponential convergence of the error system, i.e., the signals $e_1(t)$ and $e_2(t)$ both converge to zero exponentially fast if the agents do not start at the same positions (Dimarogonas and Johansson (2008); Sun et al. (2016)).

It is clear that although the error signals $e_1(t)$ and $e_2(t)$ converge to zero, the final relative positions may not guarantee any prescribed shapes as one would like to have for minimally rigid formations. In particular, agents p_1 and p_3 will lie somewhere on a circumference with the center at p_2 and the radius as d_1 and d_2 respectively. More precisely, the relative positions converge to the set

$$\mathcal{Z} \stackrel{\Delta}{=} \{z : ||z_k|| = d_k, \forall k \in \{1, 2\}\}. \tag{8}$$

Furthermore, the exponential convergence of $e(t)$ and $z(t)$ to the origin and $\mathcal{Z}$, respectively, implies that $\dot{p}_1(t)$ and $\dot{p}_2(t)$ converge to zero exponentially fast, therefore the agents converge to only one stationary shape among all the ones defined by $\mathcal{Z}$, i.e., all the agents eventually stop.