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Distributed Endogenous Internal Model for Modal Consensus and Formation Control

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Abstract: In this paper, the problems of (modal) consensus and formation control are tackled for a group of heterogeneous agents described by linear dynamics and communicating over a network with fixed topology. The classic approach to these problems prescribes that each agent is provided with its own complete internal model of the desired dynamics (which can be viewed as an exosystem). In this paper, the novel concept of *Distributed Endogenous Internal Model* is introduced and discussed. Such internal model is characterized by two features: i) it is actually *distributed over the network*, *i.e.* no single agent is provided with a complete internal model; ii) it is *endogenous*, namely it is generated by exploiting the dynamics already available to the overall group of agents, through the local cooperation between each agent and its neighbors. As a consequence, each agent is capable of generating the desired steady-state distributed static control input by only exchanging information with its neighbors, without the need for additional dynamics anywhere in the network.

Keywords: Consensus; Formation Control; Internal Model Principle; Distributed Control.

1. INTRODUCTION

The problem of characterizing as well as enforcing meaningful collective behaviors of groups of - independent and possibly heterogeneous - agents has become increasingly crucial in recent years, as it naturally arises in several contexts and applications, encompassing social sciences, biology and engineering tasks, such as mobile robots, unmanned air vehicles (UAVs) or autonomous underwater vehicles, see *e.g.* Fax and Murray [2004], Olfati-Saber and Murray [2004], Jadbabaie et al. [2003], Moreau [2005], Lin et al. [2005]. It has been immediately recognized that the role played by the information exchange pattern and the communication topology among the components of the group is of paramount importance in the resulting collective coordination and consensus, Wieland et al. [2011].

It appears evident that the interest on such coordinated motions is intrinsically related to the ability for each agent of the group to achieve individual or common desired objectives, possibly despite the limited communication and available information. Since such objectives may be naturally formulated within the framework of output regulation problems, the above problem can be indeed interpreted in terms of a distributed output regulation task for each agent, Su and Huang [2012]. This intuition essentially motivated several works, see *e.g.* Sepulchre et al. [2008], Wieland et al. [2011], De Persis and Jayawardhana [2014], that relate the solvability of the *consensus* problem - or more in general of formation control tasks - to the presence of *internal models*, Francis and Wonham [1976], of the desired (common) behavior, which must be possessed by each agent of the group. Therefore, in recent years, a large number of practical solutions have been proposed along this direction, all characterized by the fact that the internal model is essentially *embedded* into each individual agent by means of additional dynamics, while the communication topology is fundamentally employed for the stabilization task. However, in modern control applications collective motions must be obtained for extremely large-scale (even *huge*, in some circumstances) networks of agents, *e.g.* power grids, where implementing a solution that avoids replicating internal models in each individual node of the network may be a critical advantage. Moreover, the interconnection among the nodes is typically not exploited to solve the coordination task, but rather the stabilization one, hence somewhat *wasting* the behaviors that can be already achieved by considering the dynamics of each agent, without the need for introducing additional dynamics anywhere in the network. This paper aims at addressing such issues by introducing *internal models that are essentially distributed over the entire network and are not, in general, entirely possessed by any agent of the group*. The main contribution of this paper consists in introducing the concept of *distributed endogenous internal model* and in employing such notion to tackle the problems of modal consensus and formation control. The rest of the paper is organized as follows. In Section 2, we formally introduce the modal consensus and the formation control problems, together with interesting insights and notable variations. The novel notion of *distributed endogenous internal model* is the topic of Section 3. The solutions to the two problems defined above by means of distributed internal models are dealt with in Sections 4 and 5, respectively. Finally, numerical simulations, involving some interesting scenarios from the literature, are discussed in Section 6, while conclusions are drawn in Section 7.

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2. PRELIMINARIES AND PROBLEM DEFINITION

Consider a *multi-agent model* consisting of N heterogeneous linear systems described by equations

$$\dot{x}_i = A_i x_i + B_i u_i\,, \tag{1a}$$
$$e_i = C_i x_i + Q_i w\,, \tag{1b}$$

with $x_i(t) \in \mathbb{R}^{n_i}$, $u_i(t) \in \mathbb{R}^{m_i}$ and $e_i(t) \in \mathbb{R}^{p_i}$, for $i = 1, ..., N$. For later use, define [1] $\bar{n} = \sum_{i=1}^{N} n_i$, $A = \text{blkdiag}\{A_i\}$ and $B = \text{blkdiag}\{B_i\}$. The exogenous signal $w(t) \in \mathbb{R}^q$, which may be given several alternative interpretations as extensively discussed in the following, is generated by equations of the form

$$\dot{w} = Sw\,, \tag{2}$$

which is referred to as the *exosystem*, following the standard terminology used in output regulation theory. The communication topology is captured by means of a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, with vertex set $\mathcal{V} = \{v_1, ..., v_N\}$, each vertex associated to a system of the form of (1), and arc set $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$. The latter encodes the information exchange pattern, namely the i-th agent receives information from the j-th agent if and only if $\{v_j, v_i\} \in \mathcal{E}$. The adjacency matrix $\mathcal{A}$ associated to the graph $\mathcal{G}$ is constructed by letting $a_{ij} = 1$ if and only if there is an arc between v_j and v_i and $a_{ij} = 0$ otherwise. Moreover, the notation $\mathcal{N}_i$ defines the set of neighbors from which the agent i receives information. Consider in addition the matrix $L = \mathcal{A} \otimes I_{\bar{n}}$. Finally, let L_k denote the k-th block row of L, that is the $n_k \times \bar{n}$ matrix consisting of the rows from $(\sum_{j=1}^{k-1} n_j + 1)$ to n_k of the matrix L. Despite the fact that the following definitions and derivations are carried out mainly in the presence of a *time-invariant* communication topology, it is not difficult to show that a *time-varying* topology are not a critical obstruction to the extension of such concepts.

Since the notion of *steady-state response* is instrumental for the following derivations, it is briefly recalled here. Towards this end, consider first the classical decomposition of the state space in terms of the so-called stable, center and unstable subspaces, denoted by $\mathcal{V}^-$, $\mathcal{V}^0$ and $\mathcal{V}^+$, respectively. In particular, $\mathcal{V}^-$, $\mathcal{V}^0$ and $\mathcal{V}^+$ are the subspaces generated by the generalized eigenvectors corresponding to eigenvalues of the dynamic matrix with negative, zero or positive real parts, respectively. It is well-known that $\mathbb{R}^n = \mathcal{V}^- \oplus \mathcal{V}^0 \oplus \mathcal{V}^+$. Finally, let $\mathbb{C}^- \triangleq \{\lambda \in \mathbb{C} : re[\lambda] < 0\}$ and let $\bar{\mathbb{C}}^- \triangleq \{\lambda \in \mathbb{C} : re[\lambda] \leq 0\}$.

Definition 2.1. Consider an autonomous linear system $\dot{x} = Ax$ and suppose that $\sigma(A) \subset \bar{\mathbb{C}}^-$. Then $x_{ss}(\cdot)$, the *steady-state response from* x_0, is defined as

$$x_{ss}(t) = e^{At}\mathbb{P}_{\mathcal{V}^0}(x_0)\,, \tag{3}$$

where $\mathbb{P}_{\mathcal{V}^0}(x_0)$ denotes the projection of the initial condition x_0 on the subspace $\mathcal{V}^0$. ◇

It is worth noting that, differently from the classical notion of steady-state response of a *forced* dynamical system, the response $x_{ss}(t)$ in (3) depends on the *initial condition* x_0. Such definition is particularly useful in the following, where, as will be extensively discussed, the concept of distributed endogenous internal model allows the *endogenous* generation of desired signals. Two alternative formulations of the control objectives of interest, in terms of consensus or formation control, can be defined. In both cases, the resulting control law u is said to be *distributed* if, for each agent $i \in \{1, \ldots, N\}$, the input u_i depends only on information exchanged with agents belonging to the set of its neighbors $\mathcal{N}_i$.

Definition 2.2. (Modal Consensus). Consider the heterogeneous systems (1a), for $i = 1, ..., N$, and the exosystem (2). The *Modal Consensus* problem consists in determining distributed control inputs u_i, $i = 1, ..., N$ such that the steady-state responses $x_{ss,i}(\cdot)$ contain only modes of (2), and that for almost any initial condition $x(0) \in \mathbb{R}^n$ it holds that $x_{ss}(\cdot) \triangleq [x'_{ss,1}(\cdot), \cdots, x'_{ss,N}(\cdot)]'$ contains all the modes of (2). Moreover, *Strong Modal Consensus* is achieved if the steady-state response $x_{ss,i}(\cdot)$ of each agent $i = 1, ..., N$ contains all the modes of (2) for almost any initial condition $x(0) \in \mathbb{R}^n$. ◇

Remark 2.1. Note that, in Definition 2.2, the need to neglect a set of initial conditions (having zero measure) is imposed by the fact that in a linear system as (1a) there necessarily exist such a set where some natural modes are not excited. ▲

The statement of Definition 2.2 entails that the agents of the team should agree on the *modal content* (that is, the functions appearing) in their steady-state behavior; this is a weaker request with respect to the requirement in *asymptotic tracking*, where all agents are required to follow a *specific* trajectory (specified *a priori* by the exosystem's initial condition), or the requirement in *consensus*, where all agents are required to follow a *common* trajectory (not specified *a priori*). In the setting of modal consensus, the exosystem (2) is interpreted as a tool by means of which the modal content at steady-state is specified, rather than *e.g.* an actual reference generator. However, it is worth stressing that more demanding requirements can be achieved within the same framework introduced for modal consensus, essentially by considering suitable selections of the matrices S, C_i and Q_i, for $i = 1, ..., N$, as discussed in details below. In output regulation problems, it is usual to reformulate a tracking objective as the task of pushing to zero the output error e_i having the form in (1b). In the present context, in order to be able to control a formation of agents, it seems of interest to be able to have each agent follow a *phase-shifted* version of a common reference trajectory. Let, then, $w(t; w_0; t_0)$ denote the solution of (2) at time t with initial condition $w(t_0) = w_0$ and an output matrix C_w such that $C_w w$ can be seen as a reference signal, and consider the following definition.

Definition 2.3. (Formation Control). Consider the heterogeneous systems (1a) with output $y_i \triangleq C_i x_i$, for $i = 1, ..., N$, and the exosystem (2). Given $t_i \in \mathbb{R}_+$, $i = 1, ..., N$, the *Formation Control* problem consists in determining distributed control inputs u_i, $i = 1, ..., N$ such that

$$\lim_{t\to\infty} y_i(t) - C_w w(t; w_0; t_i + \Delta) = 0\,, \tag{4}$$

for some $\Delta \in \mathbb{R}$ and for all $w_0 \in \mathbb{R}^q$. ◇

Note that (4) can be reformulated as the requirement of having $\lim_{t\to\infty} e_i(t) = 0$ with e_i given by (1b), provided that Q_i is chosen as

$$Q_i = -C_w e^{S(t_i+\Delta)}. \tag{5}$$

[1] The notation $\text{blkdiag}\{A_i\}$ represents a block diagonal matrix with the matrices A_i as diagonal blocks.