ARTICLE IN PRESS

ISA Transactions xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Practice article

Robust adaptive sliding mode control for uncertain systems with unknown time-varying delay input

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ARTICLE INFO

Keywords:
Varying delay
Unknown delay
Robust control
Uncertain discrete-time system
Adaptive control
Sliding mode control
LMI

ABSTRACT

This article focuses on robust adaptive sliding mode control law for uncertain discrete systems with unknown time-varying delay input, where the uncertainty is assumed unknown. The main results of this paper are divided into three phases. In the first phase, we propose a new sliding surface is derived within the Linear Matrix Inequalities (LMIs). In the second phase, using the new sliding surface, the novel Robust Sliding Mode Control (RSMC) is proposed where the upper bound of uncertainty is supposed known. Finally, the novel approach of Robust Adaptive Sliding ModeControl (RASMC) has been defined for this type of systems, where the upper limit of uncertainty which is assumed unknown. In this new approach, we have estimate the upper limit of uncertainties and we have determined the control law based on a sliding surface that will converge to zero. This novel control laws are been validated in simulation on an uncertain numerical system with good results and comparative study. This efficiency is emphasized through the application of the new controls on the two physical systems which are the process trainer PT326 and hydraulic system two tanks.

1. Introduction

The unknown uncertainties and delays present the great problem in control of industrial systems. Although the different control methods were applied to this type of systems but there is still a big problem to solve for several researches. Among these controls, there is control by sliding mode: this type of control has attracted the attention of several researchers since the 60s thanks to its robustness in the presence of uncertainty and external disturbances [1–7,22,23,27,29]. In case of continuous system this control technique is well investigated for several types of systems: multivariable, nonlinear delayed … [1,7,10,11], but in the discrete case, we do not find more results especially in the case of systems with delay, with the exception of some articles in the linear case [5,6]. Since the Sliding Mode Control (SMC) is a robust control, many works were done on uncertain systems [7–12,20]. In Ref. [20], the author defined a sliding mode control law for a perturbed system with known and constant delay on input assuming the upper limit (or bound) of the disturbance is known.

Sometimes we know nothing about the uncertainties and disturbances of the studied systems. For this, we make recourse to the robust adaptive sliding mode control. This control technique has two phases [14–20,28]: The first is estimating unknown system parameters or information about disturbances and uncertainties (their values or their limits) and the second is the determination of the control law that ensures the sliding surface convergence towards zero.

As for the robust adaptive SMC, there are some results for systems with varying known delay on the state [19,31]. However, the case of varying unknown delay on the input is yet to be processed. In this article, we treated this case for a system with unknown and varying-time delay on entry where the uncertainties are unknown. We proposed in the first step a new sliding surface to solve the problem of unknown and varying unknown delay, where we got a reduced system that no longer depends the delay on input and stable by a condition found by applying LMIs. In the second step, we assumed that the upper bound of uncertainty is known and we proposed a novel robust sliding mode control of uncertain system with unknown and varying-time delay using the new sliding surface proposed in the step one. After we assumed the upper bound is unknown where we estimated its upper bound by offering a new robust adaptive sliding mode control law for the type of systems studied in this paper always using the new form of the proposed sliding surface.

The advantages of this new control law in relation to other works [13–17,19,20] are that, we considered the delay is unknown and variable; this is the most delicate and most general case because in the real case the delays are unknown and varying in time and the upper bound of uncertainty is unknown will be estimated. The novelty in our contribution is its possibility to be applicable on uncertain systems with delays on the control even if the delay is variable and unknown and the

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https://doi.org/10.1016/j.isatra.2018.04.017
Received 11 March 2016; Received in revised form 3 March 2018; Accepted 25 April 2018
0019-0578/ Published by Elsevier Ltd on behalf of ISA.

Please cite this article as: Benamor, A., ISA Transactions (2018), https://doi.org/10.1016/j.isatra.2018.04.017

upper bound of uncertainties are also unknown. Therefore, our work can be considered as a generalization of the robust adaptive sliding mode control for systems with delay on input. These benefits are confirmed by application on real systems: Feedback's Process Trainer PT326 and hydraulic system two tanks and comparative study with other works [20].

This paper is organized as follows: the section two starts with formulating the problem. In the third section, we present main results: we proposed new sliding surface of uncertain system with varying-time and unknown, after two new sliding mode controls (robust and robust adaptive) are defined for a class of uncertain systems with varying-time and unknown delay and we prove convergence in the finite time and stability of the new approaches, which will be validated on a digital system and real systems in fourth section.

2. Problem formulation

Consider the following uncertain discrete system with time-varying and unknown delay given by its state equation as follows:

$$\begin{cases} x(k+1) = (A+\Delta A)x(k) + Bu(k-d(k)) \\ u(k) = \psi(k) \quad k = -d_M, \quad -d_M+1, \quad \ldots, -d_m \end{cases} \tag{1}$$

Where $x \in R^n$ is the state vector, $u \in R^m$ is the input, $\psi(k)$ is a given initial condition, A and B are known real constant matrices such that $A \in R^{n\times n}$ and $B \in R^{n\times m}$, $d(k)$ is the varying time delay. ΔA is unknown uncertainty bounded matrix.

For the convenience of the Proof and without loss of generality, we can take the following assumptions:

Assumption 1. The varying time delay $d(k)$ is unknown and bounded such that $d_M \geq d(k) \geq d_m$

Assumption 2. The pair of matrices (A, B) is controllable

Assumption 3. The bounded uncertainty ΔA in (1) are assumed to have the following forms:

$$\Delta A = GF(k)H \tag{2}$$

Where G and H are known real constant matrices with appropriate dimensions and *F(k)* is unknown but norm-bounded such as $F^T(k)F(k) \leq I$.

Assumption 4. We assume, that $C\Delta Ax(k)$ is bounded as: $\|C\Delta Ax(k)\| \leq A_x$

3. Main results

In this paper, firstly, we are going to design new form of sliding surface of uncertain discrete systems. Secondly, we propose two new sliding mode control laws for a class of this system with varying delay: the first is robust where we considered the upper bound of uncertainty is known and the second is robust adaptive control where we estimated the upper bounded. Finally, we prove convergence in the finite time of two new approaches.

3.1. New form of sliding surface

In this section, we propose a sliding surface of system (1) to solve the problem of the varying-time delay.

$$s(k) = Cx(k) + \sum_{i=k-d(k)}^{k-1} CBu(i) + \sigma(k) \tag{3}$$

with:

$$C \in \mathbb{R}^{m\times n} \text{ and } \Delta\sigma(k) = \sigma(k+1) - \sigma(k) = -C[A - I_n - BK]x(k) \tag{4}$$

Where I_n is identity matrix and $K \in R^{m\times n}$.

We note:

$$\begin{aligned} \Delta s(k) &= s(k+1) - s(k) \\ &= Cx(k+1) - Cx(k) + \textstyle\sum_{i=k-d(k)+1}^{k} CBu(i) - \sum_{i=k-d(k)}^{k-1} CBu(i) \\ &\quad - C[A - I_n - BK]x(k) \\ &= Cx(k+1) + CBu(k) - CBu(k-d(k)) - C[A - BK]x(k) \end{aligned}$$

By replacing *x(k + 1)* by its expression (1), we obtained:

$$\begin{aligned} \Delta s(k) &= C(A+\Delta A)x(k) + CBu(k-d(k)) + CBu(k) - CBu(k-d(k)) \\ &\quad - C[A-BK]x(k) \\ &= C(A+\Delta A)x(k) + CBu(k) - C[A-BK]x(k) \\ &= CBu(k) + C[\Delta A + BK]x(k) \end{aligned} \tag{5}$$

The equivalent control law $u_{eq}(k)$ is derived by $\Delta s(k) = 0$, then:

$$Bu_{eq}(k) + [\Delta A + BK]x(k) = 0$$

therefore:

$$u_{eq}(k) = -B^{-1}[\Delta A + BK]x(k) \tag{6}$$

By using equation (6) the state $x(k+1)$ can be written in the following form:

$$x(k+1) = (A+\Delta A)x(k) - [\Delta A + BK]x(k-d(k)) \tag{7}$$

Which can be rewritten as:

$$x(k+1) = \widetilde{A}x(k) + \widetilde{A}_d x(k-d(k)) \tag{8}$$

with:

$$\widetilde{A} = A + \Delta A \text{ and } \widetilde{A}_d = -[\Delta A + BK] \tag{9}$$

From the above, the design of a sliding mode control:

- There exists a matrix $K \in R^{m\times n}$ which guarantees stability of system (8).
- There exists control law which makes the sliding function asymptotically stable for specified sliding manifolds.

The remainder of this sub section is devoted of sliding manifolds and control law to satisfy these requirements.

The following lemmas play an important role in the derivation of sliding surface design.

Lemma 1. [24]: *Assume that* $a \in R^{n_a}$, $b \in R^{n_b}$ and $N \in R^{n_a\times n_b}$. *Then, for any matrices* $\Phi \in R^{n_a\times n_a}$, $\Psi \in R^{n_a\times n_b}$ *and* $Z \in R^{n_b\times n_b}$ *the following inequality holds:*

$$-2a^TNb \leq \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} \Phi & \Psi - N \\ \Psi^T - N^T & \Xi \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} \tag{10}$$

where:

$$\begin{bmatrix} \Phi & \Psi \\ \Psi^T & \Xi \end{bmatrix} \geq 0$$

Lemma 2. Schur complement [25]:

We consider the following matrices $Q(x) \in R^{m\times n}$, $R(x) \in R^{n\times n}$ and $S(x) \in R^{m\times n}$ *depend on a variable* $x \in R$. *Then the following matrix inequalities:*

$$\begin{cases} R(x) > 0 \\ Q(x) - S(x)R^{-1}(x)S^T(x) > 0 \end{cases}$$

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0$$

Are equivalent., then the last matrix inequality is an LMI.