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Research article

System identification with measurement noise compensation based on polynomial modulating function for fractional-order systems with a known time-delay

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ABSTRACT

This study presents a system identification method based on polynomial modulating function for fractional-order systems with a known time-delay involving input and output noises in the time domain. Based on the polynomial modulating function and fractional-order integration by parts, the identified fractional-order differential equation is transformed into an algebraic equation. By using the numerical integral formula, the least squares form for the system identification is obtained. In order to reduce the effect of noises existing in the input and output measurements, the compensation method for the input and output noises is also studied by introducing an auxiliary high-order fractional-order system in the revised identification algorithm. Finally, the effectiveness of the proposed algorithm is verified by the simulation result of an illustrative example and the experimental result of temperature identification for a thermal system.

1. Introduction

Due to the memory property of the fractional-order calculus, fractional-order systems can reveal the dynamic behaviors of many real-world systems essentially. Hence, the study of fractional-order systems has aroused wide attention of many scholars [1]. For the historical reason, we call it a fractional-order system. In fact, fractional-order systems also cover integer-order ones, it can be said that fractional-order systems are the generalization of integer-order ones [2]. According to the analysis method of integer-order systems, the researches on the performances of fractional-order systems have emerged, such as the stability [3–5], robust stability [6,7], controllability [8] and observability [9]. Various types of fractional-order control strategies have been proposed and applied to practical applications, such as fractional-order sliding mode control [10], fractional-order active disturbance rejection control [11], fractional-order PID control [12], fractional-order iterative learning control [13], and so on.

The system identification of fractional-order systems is the foundation of the stability analysis and controller design for fractional-order systems. The system identification methods of fractional-order systems are mainly classified into two categories: the time domain method and frequency domain method. Lévy method is the basic frequency domain method [14].

In the time domain, Haar wavelet method was adopted to identify linear fractional-order systems using the input and output signals in Ref. [15]. A bias correction method of fractional-order closed-loop systems was presented to eliminate the estimation error in Ref. [16]. For nonlinear fractional-order systems, the concept of fractional-order Volterra series was proposed to provide an effective identification method in Ref. [17]. In Refs. [18,19], some intelligent optimization methods were also introduced into the identification of nonlinear fractional-order systems, but the effectiveness of these identification methods are largely dependent on the adopted optimization algorithms. For above identification methods in the time domain, the fractional-order derivative operations on the measurements of the input and output signals are required. Because the mathematical expression of the measurement signal of output is unknown, it is different to obtain the fractional-order derivatives of the output signal. Moreover, if the noise disturbance is involved in the measurement signals of the input and output, the accuracy of identification will be affected. In Ref. [20], the adjustable fractional-order differentiators were used to calculate the fractional-order derivatives of the measurement signals, and the identification

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algorithm of linear fractional-order systems was proposed. However, this algorithm consumes a lot of calculation for the identification of complex fractional-order systems. Furthermore, the identification of fractional-order systems with time-delays was discussed, and the unknown time-delay case was addressed in Ref. [21]. But, the fractional-order derivatives of the measurement signals need to be calculated.

System identification method based on the modulating function was proposed by Shinbrot in 1957 [22]. The system identification method based on the modulating function has been applied to integer-order systems in different forms of modulating functions, such as Walsh function, Hermite function, Fourier modulating function and spline-type function [23–26]. Based on thought of the identification method using the modulating function for integer-order systems, the identification methods for linear fractional-order systems under the Riemann-Liouville definition using the polynomial modulating function and Gaussian function were studied in Refs. [27] and [28] respectively. In Ref. [29], the identification of fractional-order systems under the Caputo definition was provided with the measurement noise. But the fractional-order derivatives of the measurement signal of output were not considered in Ref. [29].

Considering the consistency of initial condition requirements for fractional-order systems under the Caputo definition and integer-order systems, the Caputo definition is widely used in the descriptions of fractional-order systems and fractional-order controllers. It is of practical significance to study the identification method for fractional-order system under the Caputo definition. In this study, we mainly study the system identification method based on the polynomial modulating function proposed in Ref. [29] for fractional-order systems with a known time-delay under the Caputo definition. The properties of the modulating function and the fractional-order integration by parts, the derivatives of the measurement signals are transformed into the designed modulating function to compensate the error caused by the noises involved in the measurement signals of the input and output in the estimations of the coefficients. The dynamics behavior of a thermal system can be described by a fractional-order model with a time-delay effectively [30], hence we set up a heat conduction experimental platform to verify the effectiveness of the proposed algorithm by the sampling data in the experimental section.

The main contribution of this paper is to establish a parameter identification method for continuous-time fractional-order systems based on the sampled data, and the modulating function method is adopted in the identification method. Considering the measurement noises involved in the measurement signals of input and output, the method of noise compensation is proposed to deduce the estimation error. The model in the system identification algorithm can be applied to the fractional-order system model with the fractional-order derivatives of output signal. Considering the variances of measurement noises existing in the input and output are unknown, the calculation approach of variance estimations of measurement noises is also the innovation of this paper.

The rest of this paper is organized as follows. Section 2 introduces the definition and properties of fractional-order calculus. In Section 3, the algorithm of the system identification by using the least squares algorithm is proposed, and the calculation method of the compensation for the noises involved in the input and output measurements is proposed. The results of simulation and experiment are offered to validate the effectiveness of the proposed identification method in Section 4. Section 5 concludes this paper.

2. Preliminaries

For a function $f:[a,b]\to\mathbb{R}$, the left Riemann-Liouville (R-L) fractional-order derivative with v-order [31] is defined by

$$ {}_a^R D_x^v f(x) = \frac{1}{\Gamma(n-v)} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \int_a^x (x-\tau)^{n-v-1} f(\tau)\mathrm{d}\tau, \tag{1} $$

where $\Gamma(x)=\int_0^{+\infty} \mathrm{e}^{-t} t^{x-1}\mathrm{d}t$ is the Gamma function with the following properties $\Gamma(x+1)=x\Gamma(x)$ and $\Gamma(N)=(N-1)!$ for $N\in\mathbb{Z}^+$, $v\in\mathbb{R}^+\setminus\mathbb{Z}^+$, $n\in\mathbb{Z}^+$, and the order v satisfies $n-1<v<n$, $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^+$ is the set of positive real numbers, $\mathbb{Z}^+$ is the set of positive integer numbers.

The definition of right Riemann-Liouville fractional-order derivative with v-order is

$$ {}_x^R D_b^v f(x) = \frac{(-1)^n}{\Gamma(n-v)} \frac{\mathrm{d}^n}{\mathrm{d}x^n} \int_x^b (\tau-x)^{n-v-1} f(\tau)\mathrm{d}\tau. \tag{2} $$

The left and right Caputo fractional-order derivatives are presented respectively, by

$$ {}_a^C D_x^v f(x) = \frac{1}{\Gamma(n-v)} \int_a^x (x-\tau)^{n-v-1} f^{(n)}(\tau)\mathrm{d}\tau, \tag{3} $$

and

$$ {}_x^C D_b^v f(x) = \frac{(-1)^n}{\Gamma(n-v)} \int_x^b (\tau-x)^{n-v-1} f^{(n)}(\tau)\mathrm{d}\tau. \tag{4} $$

The relationship between the left R-L and left Caputo derivatives [32] is given as follows

$$ {}_a^C D_x^v f(x) = {}_a^R D_x^v f(x) - \sum_{k=0}^{n-1} \frac{f^{(k)}(a)}{\Gamma(k-v+1)} (x-a)^{k-v}. \tag{5} $$

Hence, ${}_a^C D_x^v f(x) = {}_a^R D_x^v f(x)$ holds if $f^{(k)}(a)=0$ for $k=0,1,\cdots,n-1$.

For the right Caputo derivative and left R-L derivative, the fractional-order integration by parts formula [33] for two functions $f(x)$ and $g(x)$ with $x\in[a,b]$ is presented as follows

$$ \int_a^b g(x)\,{}_x^C D_b^v f(x)\mathrm{d}x = \int_a^b f(x)\,{}_a^R D_x^v g(x)\mathrm{d}x + \sum_{j=0}^{n-1} (-1)^{n+j} [{}_a^R D_x^{v+j-n} g(x) \times f^{(n-1-j)}(x)] \,|_{x=a}^{b} . \tag{6} $$

3. Main results

In this section, the identification of fractional-order systems with a known time-delay is discussed by using the modulating function. In Subsection 3.1, the polynomial modulating function is designed to transform the fractional-order differential equation into an algebraic equation. In Subsection 3.2, the least squares identification method is offered based on the discretized difference equation. The compensation for the measurement noises existing in the input and the output is discussed in detail, and the identification method with the noise compensation is given in Subsection 3.3.

3.1. *Transformation of a linear fractional-order system with a known time-delay involving input and output noises*

The differential equation of the identified fractional-order system using the definition of left Caputo is defined as follows

$$ \sum_{i=1}^{N_1} a_i\,{}_0^C D_t^{\alpha_i} y(t) + \sum_{h=1}^{N_2} c_h y^{(h)}(t) + y(t) = bu(t-\delta), \tag{7} $$

$$ z(t) = y(t) + \varepsilon_y(t), \tag{8} $$

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