ARTICLE IN PRESS

ISA Transactions xxx (xxxx) xxx–xxx

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Matrix factorization based instrumental variable approach for simultaneous identification of Bi-directional path models

Benben Jiang*, Yi Luo

College of Information Science and Technology, Beijing University of Chemical Technology, Beijing 100029, China

ARTICLE INFO

Keywords:
Closed-loop identification
System modeling
System identification
Parameter estimation
Colored noise
Instrumental variable method

ABSTRACT

This paper proposes a closed-loop identification approach to integrate matrix factorization algorithms with generalized instrumental variable (GIV) techniques to simultaneously identify the parameters and orders for both forward and backward path models. Aside from the technique of UD factorization, the QR factorization technique, which possesses good numerical property, is utilized for the proposed GIV-based method. The major difficulty and novelty of the proposed approach lies in how to properly construct instruments, number of instruments, and weighting matrices to obtain enhanced identification performance. To the end, the identification accuracy properties, in terms of the covariance matrix of the parameter estimates, are provided. In addition, a sufficient condition of consistent parameter estimates for the GIV-based approach is discussed. The effectiveness of the proposed identification method is demonstrated by simulation results.

1. Introduction

The least-squares principle has been a dominant method in the field of data fitting, such as system identification and signal processing. In system identification, however, most of the existing least-squares methods identify the model orders and parameters separately [1]. One common manner is to separately identify the model parameters and loss functions for all possible orders from 0 to n, where n is the maximum possible order of the model, and then an appropriate model order is determined by examining the loss function values. Nevertheless, a deficiency for such methods is the high computational burden for large n values. In addition, another problem of the least-squares approaches is the poor numerical performance under the circumstance of ill-conditioned coefficient matrices. To the end, some alternative methods based on regularized least-squares techniques are utilized to handle this issue [20]. It is worth mentioning that regularized least-squares problems are important in the field of robotics. Other alternatives, such as matrix factorization based techniques, are also often adopted [2].

In Ref. [3], matrix decomposition techniques were investigated by incorporating a multiple model least-squares (MMLS) structure, which can solve n additional sets of linear equations without any extra computational cost. Additionally, by utilizing appropriate matrix factorization techniques, e.g., QR factorization, the MMLS approach is found to attain improved numerical performance [3]. In system identification, it shows that MMLS is superior in the aspects of information extraction and practical implementations in contrast to traditional least-squares methods [3]. The property of simultaneous estimation of additional n equation sets has specific practical meaning which corresponds to the models with orders from zero to n by virtue of appropriate formulations of data vectors. Inspired by this, Niu et al. presented a decomposition-assisted least-squares method, termed as augmented upper diagonal identification (AUDI) algorithm, to simultaneously identify the model order and parameters for open-loop systems [4,5]. By utilizing UD factorization, parameter estimates for all models with orders from zero to a sufficiently large n, as well as the corresponding loss function values, can be produced in the resulting matrices $\boldsymbol{U}$ (the parameter matrix) and $\boldsymbol{D}$ (the loss function matrix), respectively. It is worth mentioning that, in the AUDI algorithm, the physical meaning of the even columns of parameter and loss function matrices are left unclear for open-loop identification.

Many good results on the utilization of matrix factorizations for combined structure and parameter identification have also been conducted by Billings et al. [17–19]. For instance, orthogonal least squares (OLS)-based methods were presented to simultaneous structure selection and parameter estimation for nonlinear regressions, and it was found that the OLS approaches can provide a compact and powerful manner to generate parsimonious models for industrial systems [17,18].

It was reported in Refs. [21,22] that identification of processes operating under output feedback (i.e., in closed loop) is crucial for industrial applications due to safety or economic reasons. Additionally, closed-loop identification needs to be performed for the system that

* Corresponding author.
E-mail address: jiangbb@mail.buct.edu.cn (B. Jiang).

https://doi.org/10.1016/j.isatra.2018.04.018
Received 12 July 2017; Received in revised form 6 April 2018; Accepted 25 April 2018

Please cite this article as: Jiang, B., ISA Transactions (2018), https://doi.org/10.1016/j.isatra.2018.04.018

contains inherent feedback mechanisms, such as biological processes or economic systems [6,21]. Most of the closed-loop identification methods can be categorized into direct approaches, indirect approaches, and joint input-output approaches [1,21]. The direct methods often directly estimate the model of the forward path based on the measurements of input and output without considering the information of backward path. The indirect approaches firstly identify the transfer function of the whole closed-loop system and then determine the forward path model using the knowledge of the backward path. The joint input-output methods consider the inputs and outputs jointly as an augmented vector of output from a system driven by some extra inputs or set-point signals, in which the forward path model is determined from an estimation of this augmented system. However, for some systems, such as biological or economic systems, the backward paths are often unknown and are also required to be identified. Motivated by these, the problem of simultaneous determination of bi-directional path models for a closed-loop system is investigated in Ref. [6], where a novel closed-loop identification algorithm named as interleaved data pair upper diagonal (IDPUD) was presented.

The noise in most industrial processes is usually colored. The instrumental variable (IV) algorithm is an attractive tool to handle colored noise in system identification. Its main idea is to select proper instrumental variables which are independent of colored noise sequences to obtain unbiased estimation of parameters. The IV technique for system identification has received much attention for several decades. For example [7], and [8] are some early works in the engineering literature, and some more recent results have been reported in Refs. [9] [10], and [11]. In Ref. [12], Jiang et al. extend the matrix factorization-based IDPUD method by incorporating IV techniques to simultaneously estimate bi-directional path models in a closed-loop system with colored noise. It was pointed out in Ref. [8] that more accurate estimates can be obtained by utilizing generalized instrumental variable (GIV) techniques in contrast to IV techniques. However, GIV techniques are limitedly explored in the framework of matrix factorization methods for system identification.

In this paper, we investigate the matrix factorization based method in Ref. [12] in combination with GIV techniques to achieve more accurate estimates by incorporating particular design variables (i.e., instruments, number of instruments, and weighting matrices). Compared to [12], the major difficulty and novelty of the proposed GIV-IDPUD method lies in how to construct the design variables properly to improve the identification performance. Moreover, aside from the technique of UD factorization, the QR factorization technique, which possesses good numerical property, will be utilized for the proposed approach. The consistency property is an important aspect for closed-loop identification methods. The discussion on the consistency of the proposed GIV-IDPUD approach will be provided, while the consistency property of identification algorithms was not investigated in Ref. [12]. The original idea of the proposed method was briefly stated in Ref. [14].

The rest of this paper is organized as follows. In Section 2, the investigated problem is formally stated. The GIV-IDPUD algorithm is proposed for the ARMAX process in the presence of colored noise in Section 3. Section 4 discusses the consistency and identification accuracy properties of GIV-IDPUD. The effectiveness of the proposed algorithm is demonstrated by numerical examples in Section 5, followed by concluding remarks in Section 6.

2. Problem formulation

A closed-loop process with two data collection ends is considered, as given in Fig. 1. The forward and backward paths with x and y being the corresponding outputs are modeled by the following linear time-invariant systems (1)

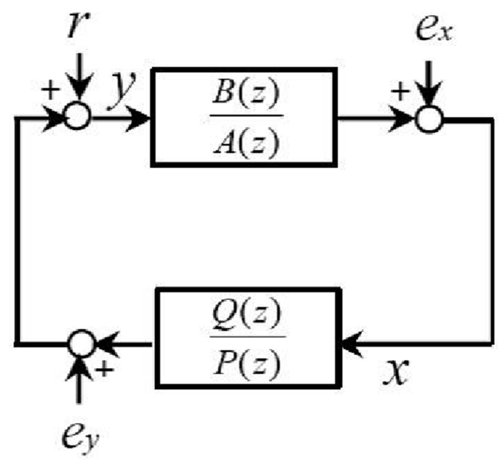

Fig. 1. Block diagram of the closed-loop system as modeled in (1a) and (1b).

$$x(k) = \frac{B(z)}{A(z)}y(k) + e_x(k), \tag{1a}$$

$$y(k) = \frac{Q(z)}{P(z)}x(k) + r(k) + e_y(k), \tag{1b}$$

where

$$\begin{cases} A(z) = 1 + a_1 z^{-1} + \cdots + a_{n_x} z^{-n_x}, \\ B(z) = b_d z^{-d} + \cdots + b_{n_x} z^{-n_x}, \\ P(z) = 1 + p_1 z^{-1} + \cdots + p_{n_y} z^{-n_y}, \\ Q(z) = q_c z^{-c} + \cdots + q_{n_y} z^{-n_y}. \end{cases} \tag{2}$$

In the above expressions, Eqs. (1a) and (1b) denote the models of the forward and backward paths, respectively. z is the forward shift operator, and a_j, b_j , p_j and q_j denote the model parameters. Integers n_x and n_y are the corresponding orders, and nonnegative integers d and c represent the delays in the forward and backward paths, respectively. Variables e_x and e_y represent colored noise sequences. Variable r denotes an external signal assumed to be uncorrelated with e_x and e_y.

Assumption 1. [13]. There exists a delay somewhere in the loop, that is, $\lim_{z\to\infty} \frac{B(z)}{A(z)} \cdot \lim_{z\to\infty} \frac{Q(z)}{P(z)} = 0$ or $c + d > 0$.

Assumption 2. [13,14]. Process noise e_x and e_y are uncorrelated, that is, $e_x(i) \perp e_y(j)$, for any i and j.

The problem of simultaneous identification of both the forward and backward paths for the closed-loop process with colored noise in (1a) and (1b) is investigated in this paper.

3. Generalized instrumental variable approach based on matrix factorization for closed-loop identification

In this section, we first formulate a multiple model structure associated with the forward and backward path models with orders from 0 to a sufficiently large n, and then construct information matrices with interleaved form by the aid of generalized instrumental data vector. By performing UD- or QR-factorization on the constructed interleaved matrices, we can simultaneously obtain all the parameter estimates and the corresponding loss function values for both forward and backward path models with orders from zero to n.

3.1. Multiple model structure

Define an interleaved data vector as

$$\boldsymbol{\varphi}(k) = [-x(k-n), y(k-n), \cdots, -x(k), y(k)]^{\mathrm{T}}, \tag{3}$$