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Research article

Fractional order uncertainty estimator based hierarchical sliding mode design for a class of fractional order non-holonomic chained system

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ABSTRACT

This paper proposes a novel fractional order sliding mode control approach to address the issues of stabilization as well as tracking of an N-dimensional extended chained form of fractional order non-holonomic system. Firstly, the hierarchical fractional order terminal sliding manifolds are selected to procure the desired objectives in finite time. Then, a sliding mode control law is formulated which provides robustness against various system uncertainties or external disturbances. In addition, a novel fractional order uncertainty estimator is deduced mathematically to estimate and mitigate the effects of uncertainties, which also excludes the requirement of their upper bounds. Due to the omission of discontinuous control action, the proposed algorithm ensures a chatter-free control input. Moreover, the finite time stability of the closed loop system has been proved analytically through well known Mittag-Leffler and Fractional Lyapunov theorems. Finally, the proposed methodology is validated with MATLAB simulations on two examples including an application of fractional order non-holonomic wheeled mobile robot and its performances are also compared with the existing control approach.

1. Introduction

Generally, the mechanical systems encounter various kinds of constraints and restrictions in their motions. Mathematically, these constraints or restrictions can be represented in the form of their positions or velocities. Fundamentally, the non-integrable constraints imposed on the velocities of a mechanical system are labeled as non-holonomic constraints. The term ‘non-holonomic system’ was introduced by Hertz in 1894 [1] which consists of non-holonomic constraints such as sliding and rolling motions. It has also been pointed out that there is a strong connection between these systems and non-linear control theory which can be illustrated as in Ref. [2]: 1. These systems cannot be stabilized via continuous or smooth time invariant control laws due to violation of the Brockett's necessary conditions of stability theorem [3,4]. 2. The non-holonomic constraints are described by the non-integrable distributions, in which the bracket of any two vector fields may not generate a vector field in the same distribution which prompts us to exploit the non-linear control theory. In order to implement the control laws for these systems, the complex structure of the non-holonomic systems has been simplified by Murray and Sastry's chained architecture [5]. Various control approaches including variable structure control [6], continuous time varying control laws [7,8], hybrid control laws [9], adaptive method [10] etc. have been proposed in literature to stabilize such systems. But, all these proposed controls have been developed only for the integer order non-holonomic systems, not for their fractional counterparts.

Fractional calculus has already upgraded the theories related to modeling and control techniques [11]. Nevertheless, the notions of fractional calculus model the actual physical systems with the variable fractional derivative with greater flexibility and accuracy as compared to the integer order calculus. Fractional versions of integer order systems such as affine form of fractional order non-linear systems, chaotic systems [12], non-holonomic systems [13,14] etc. have been deeply investigated by the research community. Further, the control schemes such as fractional order PID [15], fractional order sliding mode control [16] etc. have also been established in literature for the integer order as well as fractional order applications. In addition, the Mittag-Leffler and Fractional Lyapunov results [17] have been utilized to determine the asymptotic stability of the fractional order systems.

Among the aforementioned control schemes, the sliding mode control (SMC) is one of the most powerful robust control

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techniques developed for providing robustness against various kinds of system uncertainties or external disturbances. Mathematically, the reaching and sliding laws formulate a complete robust control law [18]. However, chattering and infinite time convergence are the challenges of the first order SMC (integer as well as fractional order). The methods in Refs. [12,19] have been proposed to get rid of chattering in fractional order SMC caused due to infinite switching action. Finite time stabilization is regarded as the main priority for the control designers due to the fast response time and other high time domain performances [20]. In Ref. [21], the terminal SMC law is established for finite time convergence of system dynamics to equilibrium and faces the control singularity issue which is tackled by the non-singular terminal SMC presented in later research [22]. In Ref. [23], the hierarchical fractional order terminal SMC is devised for stabilization and tracking of fractional order non-linear systems. Similar sliding surfaces have been utilized in this paper to accomplish our prescribed objectives in finite time.

Research contributions in Refs. [24–30] have been presented with SMC for integer order non-holonomic systems. The authors in Ref. [24] have devised a finite time controller for the extended chained form of nominal non-holonomic systems in which the virtual controls are initially designed through which the actual control actions are computed. In Ref. [25], a finite time recursive terminal SMC is designed for their stabilization by initiating with the formation of two subsystems. Moreover, a disturbance observer based sliding mode control has been implemented for the third [26] and n-dimensional non-holonomic systems [27] by using [28] to achieve the finite time tracking objective. Fast terminal SMC approach and adaptive approaches have also been employed in Refs. [29] and [30], respectively.

However, there are certain research gaps in the previously addressed methods, which are as follows: Firstly, most of these methods have not considered system uncertainties or external disturbances which undoubtedly prevail in any mathematical model of a physical system. Secondly, these developed control schemes considering the system uncertainties demand their upper bounds which are not known to the designer practically. Thirdly, the exponentially or asymptotically stable controllers designed for these systems allow the convergence of the system dynamics in infinite time. Fourthly, all these prior works deal with the integer order non-holonomic systems, but not with their fractional counterparts which provide more accurate models.

In Ref. [13], the authors have derived a control scheme using three step procedures for stabilization of the fractional order non-holonomic systems in the presence of the system uncertainties. But, in this method, the upper bounds of the uncertainties have to be known in initial phase, which may not be practically available. Moreover, in this published work, no attention has been given to track the reference model. However [14], presents a fractional order SMC design for tracking objective of the same class of systems, but, subjected to known system uncertainties. Thus, in this paper, we have added an auxiliary control based on uncertainty and disturbance estimator (UDE) to the equivalent sliding mode control. This estimator needs a low pass filter with sufficient bandwidth for mitigating the system uncertainties. The estimator was originally introduced in Ref. [32] for LTI systems and it has also been employed for sliding mode control of perturbed linear plants [33]. The authors in Ref. [34] have implemented UDE based continuous control for the non-affine non-linear plants.

Motivated by the prior works, we have derived a fractional order uncertainty estimator especially for the fractional order non-holonomic systems which does not ask for the upper bounds of system uncertainties. Such a work has not been reported in the literature. The highlights of the paper are as follows: The approach developed in this paper fills all the above addressed research gaps by introducing a novel control algorithm used to stabilize as well as track the fractional order non-holonomic systems in the presence of system uncertainties as well as external disturbances. The hierarchical sliding manifolds are selected properly and then the robust control law is derived by using Mittag-Leffler and Fractional Lyapunov method. In addition, an analytically derived fractional order disturbance estimator won't need any upper bounds on these uncertainties. Our method is applicable for N-dimensional fractional order non-holonomic systems. Chatter-free control law is also guaranteed with the proposed method as the discontinuous action is not needed. Two examples of fractional order non-holonomic system with extended chain form including an engineering application of wheeled mobile robot have been utilized to validate the proposed method using MATLAB simulations. Moreover, the proposed approach has also been compared with the recently presented method in Ref. [14].

This paper is organized as: In Section 2, the basics of fractional calculus are presented. In Section 3, the control problem is stated. Section 4 presents the main novel control methodology. Section 5 shows the MATLAB simulations and their results of implementation. Section 6 demonstrates the concluding remarks.

2. Elementary knowledge about fractional calculus

Fundamentally, three kinds of definitions are described in literature for fractional calculus: Riemann Liouville (RL), Grunwald Letnikov (GL) and Caputo Derivatives. The RL derivative requires fractional order initial conditions which may not deliver a clear physical significance, thereby, making it impossible to measure for several engineering applications. The GL derivative produces slightly inaccurate results during the initial phase of simulation. However, the Caputo Derivative utilizes the clear meaningful initial conditions identical to the integer order calculus and therefore, useful in modeling, control and analysis [35]. Thus, this paper also utilizes the Caputo derivative definition.

Definition 1. The q^{th} order integration of time varying function$h(t)$ is given as [35]:

$$I^q h(t) = \frac{1}{\Gamma(q)} \int_{t0}^{t} \frac{h(\upsilon)}{(t-\upsilon)^{-q+1}} d\upsilon \tag{1}$$

where $\Gamma(\cdot)$denotes the Gamma function.

Definition 2. The q^{th} order Caputo derivative of any function $h(t)$is defined as [35]:

$$D^q h(t) = \begin{cases} \dfrac{1}{\Gamma(k-q)} \displaystyle\int_{t0}^{t} \frac{h^{(k)}(\upsilon)}{(t-\upsilon)^{-k+q+1}} d\upsilon, k-1<q<k; \\ \dfrac{d^k h(t)}{dt^k}, q = k \in Z^+ \end{cases} \tag{2}$$

Theorem 1. Consider the equilibrium $x = 0$of q^{th} order non-linear system [17]:

$$D^q x(t) = h(t,x) \tag{3}$$

and if a Lyapunov function $V(x,t)$satisfies the following conditions:

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