ISA Transactions xxx (2018) 1–12

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Chaos synchronization of uncertain chaotic systems using composite nonlinear feedback based integral sliding mode control

Saleh Mobayen

Electrical Engineering Department, University of Zanjan, P.O. Box 38791-45371, Zanjan, Iran

ARTICLE INFO

Article history:
Received 17 March 2016
Received in revised form
3 March 2018
Accepted 29 March 2018
Available online xxx

Index Terms:
Chaos synchronization
Integral sliding mode
Linear matrix inequality
Composite nonlinear feedback
Time-delay

ABSTRACT

This paper proposes a combination of composite nonlinear feedback and integral sliding mode techniques for fast and accurate chaos synchronization of uncertain chaotic systems with Lipschitz nonlinear functions, time-varying delays and disturbances. The composite nonlinear feedback method allows accurate following of the master chaotic system and the integral sliding mode control provides invariance property which rejects the perturbations and preserves the stability of the closed-loop system. Based on the Lyapunov- Krasovskii stability theory and linear matrix inequalities, a novel sufficient condition is offered for the chaos synchronization of uncertain chaotic systems. This method not only guarantees the robustness against perturbations and time-delays, but also eliminates reaching phase and avoids chattering problem. Simulation results demonstrate that the suggested procedure leads to a great control performance.

1. Introduction

The nonlinear systems, which model our real world, can demonstrate a variety of features containing chaos and hyper-chaos [1,2]. A chaotic system is a nonlinear dynamic system which sensitively depends on the initial conditions. In the last decades, chaos theory has received much attention in the areas of semiconductor lasers, information security, process control, salt-water oscillators, cryptography, data transmission, secure communication, power electronics, biological systems, chaotic finance system and so on [3–5]. Chaos synchronization of different chaotic systems has been one of the major control approaches which are extensively discussed for several years. The chief idea of synchronization between two chaotic systems is to design an appropriate control technique to force the states of the slave system track those of the master system. In the past years, various control techniques such as adaptive control [6], active control [7], linear feedback method [8], passive control [9], backstepping design [10], stochastic control [11,12], optimal control [13], sliding mode control (SMC) [14], impulsive method [15] and fuzzy logic control [16] have been developed for chaos synchronization of the chaotic systems. Among the stated methods, SMC has some special characteristics such as robustness against parametric uncertainties, satisfactory transient response, computational simplicity and less sensitivity to bounded perturbations [17].

In Ref. [18], an active SMC procedure is suggested for synchronization of two uncertain chaotic systems with parametric uncertainties in linear and nonlinear parts of the system dynamics. In Ref. [19], a robust fuzzy SMC method is planned for synchronization of two chaotic nonlinear gyros with uncertainties and external disturbances. In Ref. [20], an SMC synchronization control approach based on RBF neural network is suggested for two chaotic systems in the presence of nonlinear uncertainties and exterior disturbances. In Ref. [21], a chattering-free SMC technique with both integral and differential operators is proposed for chaos control and synchronization of uncertain nonlinear chaotic systems. A robust adaptive SMC scheme is suggested in Ref. [22] to realize chaos synchronization between two different chaotic systems in the presence of parameter uncertainties, fully unknown parameters and external disturbances. In Ref. [23], the problem of chaos synchronization between uncertain chaotic systems with a diverse structure is studied using a second-order SMC method. In Ref. [24], a discrete-time SMC technique is proposed for uncertain master–slave chaotic synchronization systems. The above-mentioned works have accelerated the progress to improve the procedure of the chaos synchronization. Nevertheless, more researches on the control methodologies need to be conducted to achieve better results and higher performance for the chaos synchronization of

E-mail address: mobayen@znu.ac.ir.
URL: http://www.znu.ac.ir/members/mobayen_saleh

https://doi.org/10.1016/j.isatra.2018.03.026

chaotic systems. To the best of our knowledge, no work has been developed on the transient performance improvement for the synchronization of the uncertain chaotic systems using the combination of composite nonlinear feedback (CNF) and integral sliding mode control (ISMC) methods, which is still open in the literature. This motivates the current research.

The CNF method was primarily proposed by Lin et al. [25] for a class of second-order systems with state-feedback. CNF has been developed to improve the transient performance of the system by presenting a nonlinear feedback law [26,27]. The CNF controller consists of linear and nonlinear control laws without any switching element [28–30]. The linear feedback law is designed to yield small damping ratio which achieves quick response. The nonlinear feedback law is designed to slowly change the damping ratio of the system as the output of the response system approaches that of the reference system so as to reduce the overshoot caused by the linear controller [28]. It is clear from the structure of the CNF control law that the obtained controller reduces to a linear controller when the gains of the nonlinear part vanish [31]. On the other hand, for the improvement of the robustness performance, the ISMC approach has been developed [32,33]. In the ISMC method, by adding an integral term to the conventional sliding surface, the reaching phase of SMC is eliminated and the trajectories of the system start moving on the proposed surface right from any initial conditions [34,35]. Actually, the idea of ISMC focuses on the system robustness in the entire state space rather than only in the sliding phase [36]. The order of the motion equation in ISMC method is equal to the dimension of the state space. Therefore, the robustness performance can be guaranteed through the entire response of the system. Due to its capability, ISMC has received a lot of consideration and it has also been proposed for matched and unmatched uncertain systems [37].

In comparison with the former studies, any investigation has been proposed the combination of composite nonlinear feedback control design and integral sliding mode control for the transient performance improvement and fast and accurate chaos synchronization of uncertain chaotic systems with Lipschitz nonlinear functions, time-varying delays and parametric disturbances. In this paper, we consider the chaos synchronization problem for a class of uncertain chaotic systems with multiple time-varying delays, Lipschitz nonlinearities, parametric uncertainties and disturbances. We recommend the theory of the combination of CNF and ISMC techniques for asymptotic chaos synchronization, improvement of the synchronization performance and guaranteeing the invariance property in perturbation rejection. Moreover, using the Barbalat lemma and Lyapunov–Krasovskii stabilization theory, the asymptotic stability conditions are derived in the form of linear matrix inequalities (LMIs). The main contributions of this paper are listed as follows:

- Combination of CNF scheme and ISMC for robustness against perturbations, transient performance improvement and accurate chaos synchronization.
- Satisfaction of the asymptotic stability conditions in the form of LMIs via Barbalat lemma and Lyapunov–Krasovskii theory.

The rest of the paper is organized as follows. Section 2 provides the mathematical model of the chaotic systems and some preliminaries. In Section 3, the theory of the combination of CNF and ISMC methods for the chaos synchronization is proposed and the asymptotic synchronization conditions are analyzed based on Lyapunov-Krasovskii stability theory. The simulation results of the Chua's chaotic system are shown in Section 4 and the conclusion is drawn in Section 5.

2. Problem description and preliminaries

Consider a class of chaotic systems as follows:

$$\begin{aligned} \dot{x} &= A_1 x + A_2 f(x), \\ y &= Cx, \end{aligned} \tag{1}$$

where $x \in R^n$ is the state vector, $y \in R$ is the output variable, and $f(x)$ is the nonlinear function which satisfies the Lipschitz condition. The matrices A_1, A_2 and C denote known constant matrices. Consider the following form of the master chaotic system:

$$\begin{aligned} \dot{x}_m &= A_{1_m} x_m + A_{2_m} f(x_m), \\ y_m &= C_m x_m, \end{aligned} \tag{2}$$

where $x_m \in R^n$ and $y_m \in R$ are the state and output of the master system. The matrices A_{1_m}, A_{2_m} and C_m are constant matrices. The slave chaotic system is assumed as follows:

$$\begin{aligned} \dot{x}_s &= \left(A_{1_s} + \Delta A_{1_s}(r)\right) x_s + \sum_{i=1}^{N} A_{d_i} x_s(t-\tau_i) \\ &\quad + A_{2_s} f(x_s) + (B + \Delta B(s))u + W(q), \\ y_s &= C_s x_s, \end{aligned} \tag{3}$$

where $x_s \in R^n$, $u \in R^n$ and $y_s \in R$ denote the states, control input and output of the slave system, respectively. The matrices A_{1_s}, A_{2_s}, A_{d_i} and C_s signify some constant matrices with appropriate dimensions. The terms $\Delta A_{1_s}(\cdot)$ and $\Delta B(\cdot)$ specify uncertainties of the system, and the vector function $W(q) \in R^n$ is the external disturbance. The uncertain parameters $(r, s, q) \in \aleph$ are Lebesgue measurable, where $\aleph$ is the compact bounding set.

Assumption 1. The state variable x_m with $\|x_m\| \le \kappa$ is the bounded state of the master system, where κ is a positive constant. There exist two matrices G and H with suitable dimensions sustaining [38]:

$$\begin{aligned} \begin{bmatrix} A_{1_s} & B \\ C_s & 0 \end{bmatrix} \begin{bmatrix} G \\ H \end{bmatrix} &= \begin{bmatrix} GA_{1_m} \\ C_m \end{bmatrix}, \\ A_{2_s} &= GA_{2_m}. \end{aligned} \tag{4}$$

Assumption 2. [39]: The continuous nonlinear function $f(x)$ fulfills the Lipschitz condition for all $x \in R^n$ and $y \in R^n$, that is:

$$\|f(x) - f(y)\| \le \wp \|x - y\|, \tag{5}$$

where $\wp$ is the Lipschitz constant. The inequality (5) can be illustrated as:

$$(f(x) - f(y))^T (f(x) - f(y)) \le \wp^2 (x-y)^T (x-y). \tag{6}$$

Assumption 3. The time-delay matrix A_{d_i} is matched, viz.:

$$A_{d_i} = BN_{d_i}, \tag{7}$$

where N_{d_i} denotes the constant matrix with suitable dimension.

Assumption 4. There exist continuous bounded perturbation functions as $N_s(\cdot)$, $M(\cdot)$ and $L(\cdot)$ with suitable dimensions so that [38]: