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Practice article

Persistently-exciting signal generation for Optimal Parameter Estimation of constrained nonlinear dynamical systems

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ABSTRACT

This work presents a novel methodology for Sub-Optimal Excitation Signal Generation and Optimal Parameter Estimation of constrained nonlinear systems. It is proposed that the evaluation of each signal must also account for the difference between real and estimated system parameters. However, this metric is not directly obtained once the real parameter values are not known. The alternative presented here is to adopt the hypothesis that, if a system can be approximated by a white box model, this model can be used as a benchmark to indicate the impact of a signal over the parametric estimation. In this way, the proposed method uses a dual layer optimization methodology: (i) Inner Level; For a given excitation signal a nonlinear optimization method searches for the optimal set of parameters that minimizes the error between the outputs of the optimized and benchmark models. (ii) At the outer level, a metaheuristic optimization method is responsible for constructing the best excitation signal, considering the fitness coming from the inner level, the quadratic difference between its parameters and the cost related to the time and space required to execute the experiment.

1. Introduction

Obtaining a suitable mathematical model of Nonlinear Dynamic System (NDS) — the plant model — is of fundamental importance to both synthesis and tuning of any robust model-based observer or controller [1]. Furthermore, the model is necessary for carrying out both the (analytical) stability assessment and the (numerical) system performance assessment [2–5]. Such a light-gray-box model — differential equations with parameter estimation — has to satisfactorily reproduce the dynamical behavior of a plant.

In this way, parameters estimation of nonlinear systems has been extensively investigated in the literature as described in Refs. [6–10]. There are several methods proposed in the literature such as optimization techniques, neural networks [11], Fuzzy [12], and others [13]. A full description of these methods, as well as their limitations and advantages, can be found in Ref. [14].

However, one common drawback for any Optimal Parameter Estimation (OPE) methodology is the necessity of an input signal that presents some properties such as: (i) its generation must be trivial, (ii) it must consider boundaries and safety, and (iii) it must provide a rich excitation to estimate the system's dynamics [8,14–16]. Due to these facts it is common that the Optimal Parameter Estimation (OPE) techniques are coupled with Optimal Input Design (OID).

As for the problem of combined Optimal Input Design (OID) with Optimal Parameter Estimation (OPE), several contributions have been made to the literature since the early 1960's [15,17,18]. It has been shown that step-like signals such as (Amplitude-modulated) Pseudo-Random Binary Signal((A)PRBSs) [4,5] are much richer than the sum of a multitude of sinusoidal signals, therefore more likely being persistently exciting [5,17]. Sufficiently informative data can be raised through the use of persistently exciting signals of appropriate order [19]. It has also been shown that it is always better to excite all the inputs simultaneously [19,20], either in open- or closed-loop parameter estimation frameworks.

Reference [21] shows a Lagrangian-based optimization methodology but it only directly applies constraints to the input variables, the output is constrained by predicting the maximum input space envelope without violations. In rigid body dynamics the computa-

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tional complexity of such approach would be inviable.

There are other references that deal with Optimal Input Design [22–24] with constraints only over the body frame. Their focus is the system's operational limits; they do not consider conditions such as minimum space and time to run de experiment. In fact, to the best of our knowledge, there is no work in literature for such purpose. However, this is an important aspect in situations where tests must be carried out in restricted spaces or considering plants that demands a considerable amount of time and money to be deployed and tested, such as deep water Remotely Operated Vehicles (ROVs) [25].

Another important observation is that those works approximate the non-differentiable input signals into a series of differentiable functions in order to use gradient-based optimization techniques. However, from an optimization perspective [26,27], to turn discrete variables into continuous ones tends to provide poor results.

Under the aforementioned background this work presents a new approach for generating persistently-exciting signals for Optimal Parameter Estimation (OPE) of constrained NDSs. It is based on the fact that, ideally a methodology should be able to start by finding the best parameter set that minimizes the output error between a parametric optimized model and the real system. This result would determine a Light-Gray Box Model (LGBM) of the real system. However, as stated before, although the excitation signal has a major impact over the correct parameter estimation, it is not possible to directly evaluate how good a resulting Light-Gray Box Model (LGBM) represents the real system.

Therefore, an important contribution of this work is the formulation of the following hypothesis; "*if it is possible to generate a White Box Model (WBM) of a nonlinear system, it is also possible to use this White Box Model (WBM) as the benchmark and evaluate if a given signal is rich enough to estimate the desired parameters. If so, the same signal may be used on the real system to estimate its parameters*". This hypotheses assumes that if a system can just be reasonably modeled as a White Box Model (WBM), its behavior — even not perfectly — may represent the real system dynamics. By adopting this hypothesis, it is now possible to use a White Box Model (WBM), which has well known parameters, as a benchmark to emulate the real system. Now, it would be also possible to use an optimized Light-Gray Box Model (LGBM) to evaluate a given signal by comparing the resulting parameters with those of the benchmark systems. This approach would be able to provide both excitation signal and the system parameters. Furthermore, it would be also possible to include constraints in the optimization formulation that would allow to keep the system under desirable limits during the real world experiment.

The solution consists of a dual layer optimization strategy where the inner layer is responsible for finding the best parameter set that minimizes the output error between the optimized Light-Gray Box Model (LGBM) and the benchmark White Box Model (WBM) system. The outer layer is responsible to evaluate the effectiveness of each signal used at the inner layer by using three metrics: a) the error fitness provided by the inner layer, b) space constraint, safety and cost to execute the signal, and c) the parametric error between the optimized and real system.

Due to the problem characteristics, its solution adopts two different optimization algorithms. Regarding the inner layer process, although literature shows several works using Least Square [28–31] to estimate the parameters, this work uses the *Safety Barrier Interior Point algorithm* [32] due to its superior mathematical stability and faster convergence time. As any other Lagrangian-based method, it enables the analysis of both Lagrange multipliers and slack variables. As for the outer layer, it is responsible for generating and evaluating the input signal through a multiple-criteria penalty function composed by the metrics mentioned above. For this stage literature shows that dynamic optimization [33] and population-based methods such as particle swarm optimization (PSO) [34]. This paper will also use a PSO-like algorithm adding three different evolutionary operators in order to avoid local-optima. This strategy is based on [35].

To demonstrate the proposed framework, this work is divided in two parts. The first part is dedicated to the problem formulation and the presentation of a practical tutorial Case. The second part focus is on sensibility, convergence and an interactivity study through the verification and discussion of several scenarios.

This paper consists of the first part, and it is organized as follows; the concepts, ideas and propositions are presented in Section 2; the metrics and the inner layer are shown in Section 3; the outer layer optimization methodology is shown in Section 4; the results in Section 5; and the conclusions in Section 6.

2. Suboptimal Excitation Signal Generation and Optimal Parameter Estimation (SESGOPE)

2.1. Preliminary discussion

Consider that the real system $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma})$ is an underwater robot such as [25,36] and must be approximated by a non-linear parametric model $\boldsymbol{\mathcal{M}}(\widehat{\boldsymbol{\Gamma}})$ where $\boldsymbol{\Gamma}$ is the set of real and unknown parameters and $\widehat{\boldsymbol{\Gamma}}$ is its estimation. Consider also that $\boldsymbol{y_r}$ and $\boldsymbol{y_m}$ are the output signal histories from the real and modeled systems respectively. The estimation of $\widehat{\boldsymbol{\Gamma}}$ can be done by using an excitation signal $\boldsymbol{u}$ over $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma})$ and trying to minimize $\|\boldsymbol{y_r} - \boldsymbol{y_m}\|$. However, several considerations must be made. For example, if the experiment must be carried out in a closed environment, then that $\boldsymbol{u}$ must be carefully designed in order to keep the desired operating limits such as velocity, position, depth, etc. Moreover, $\boldsymbol{u}$ must also encode the correct frequency and amplitude response to estimate $\widehat{\boldsymbol{\Gamma}}$. For instance, if the frequency of $\boldsymbol{u}$ is much higher or lower than the one that drives the dynamics of $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma})$, the parameters will not be well estimated [8,14]. Similarly, if the signal amplitude is not properly chosen, non-linearities such as thruster saturation, viscous friction and added mass could not be captured.

The problem, then, is to find a signal $\boldsymbol{u}$ that is able to correctly identify all specified parameters while keep the experiment under desirable operating limits. However, even though it is possible find a signal $\boldsymbol{u_i}$ that allows to find a parameter estimative $\widehat{\boldsymbol{\Gamma}}_{\boldsymbol{i}}$ that keeps all desired limits, it is not possible to ensure that $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma}) \equiv \boldsymbol{\mathcal{M}}(\widehat{\boldsymbol{\Gamma}}_{\boldsymbol{i}})$ for other signals. Moreover, the only way to test it would be using several other signals, which is inviable in most real cases.

Alternatively, it is possible to design a white box model $\boldsymbol{\mathcal{M}}(\widehat{\boldsymbol{\Gamma}}^{-})$ from $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma})$, where $\widehat{\boldsymbol{\Gamma}}^{-}$ is an initial parameter estimation and use this model as a benchmark system. Thus, it is also possible to use a signal $\boldsymbol{u_i}$ and obtain an estimative $\boldsymbol{\mathcal{M}}(\widehat{\boldsymbol{\Gamma}}_{\mathbf{i}})$ that minimizes the error between the output of both estimated and benchmark systems. Moreover, it is also possible to design an optimization problem where the objective function is, for instance, minimize $\|\widehat{\boldsymbol{\Gamma}}_{\mathbf{i}} - \widehat{\boldsymbol{\Gamma}}^{-}\|$ and the constraints are time and desirable operational limits. Finally, if $\boldsymbol{u_i}$ can be used to correctly estimate $\widehat{\boldsymbol{\Gamma}}^{-}$, then it should be also good for estimating the parameters of $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma})$.

2.2. Mathematical presentation

Consider the an (NDS) $\boldsymbol{\mathcal{R}}(\boldsymbol{\Gamma})$ that can be satisfactorily approximated by a nonlinear parametric model $\boldsymbol{\mathcal{M}}(\widehat{\boldsymbol{\Gamma}})$ with n states, p inputs, m outputs, and r parameters, which is defined as

$$\boldsymbol{\mathcal{M}}(\widehat{\boldsymbol{\Gamma}}) := \begin{cases} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t), \widehat{\boldsymbol{\Gamma}}) \\ \boldsymbol{y}(t) &= \boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t), \widehat{\boldsymbol{\Gamma}}) \end{cases} \tag{1}$$

with initial state $\boldsymbol{x_0} = \boldsymbol{x}\ (0)$, where $\boldsymbol{x}(t) \in \mathbb{R}^n \,|\, \boldsymbol{x}(t) = [\,x_1(t),\ x_2(t),\ \ldots,\ x_n(t)\,]^T$ is the state vector, $\boldsymbol{u}(t) \in \mathbb{R}^p \,|\boldsymbol{u}(t) = [\,u_1(t),\ u_2(t),\ \ldots,\ u_p(t)\,]^T$ is the input vector, $\boldsymbol{y}(t) \in \mathbb{R}^m \,|\, \boldsymbol{y}(t) =$

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