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Switched systems based on unstable dissipative systems

E. Campos-Cantón *

* División de Matemáticas Aplicadas, Instituto Potosino de Investigación Científica y Tecnológica A.C., Camino a la Presa San José 2055 col. Lomas 4a Sección, 78216, San Luis Potosí, SLP, México. (e-mail: eric.campos@ipicyt.edu.mx)

Abstract: In this work we present an unified theory of how to yield multi-scroll chaotic attractors based on step function, saturation, hysteresis and deterministic Brownian motion. This class of systems is constructed with a switching control law by changing the equilibrium point of an unstable dissipative system. The switching control law that governs the position of the equilibrium point changes according to the number of scrolls that is displayed in the attractor.

Keywords: Chaos theory; switching functions; system design; piecewise linear analysis; multi-scroll attractors.

1. INTRODUCTION

Switched systems have been widely used in many different areas in science. Some recent analysis have been made regarding their stability (See Chiou *et al.* (2010); Ma & Zhao (2010); Aleksandronov *et al.* (2011), and the references therein). There is some interest in generating chaotic or hyperchaotic attractors with multiple scroll with this kind of systems. Since the work reported by Suykens & Vandewalle (1993) about n-Double scroll from the Chua's system(Chua *et al.*, 1986; Madan , 1993), there have been many different approaches to yield multi-scroll attractors in the last coupled decades. These approaches may be ranged from modifying the Chua's system by replacing the nonlinear part with different nonlinear functions (Suykens & Vandewalle, 1993; Suykens *et al.*, 1997; Yalçin *et al.*, 2000; Tang *et al.* , 2001), to the use of nonsmooth nonlinear functions such as, hysteresis (Lü *et al.* , 2004 A; Deng & Lü, 2007), saturation (Lü *et al.*, 2004 B; Sánchez-López *et al.*, 2010), threshold and step functions (Lü, Murali *et al.*, 2008; Elwakil *et al.*, 2000; Yalçin *et al.*, 2002; Yu *et al.*, 2005; Lü *et al.*, 2003; Qiang & Xin, 2006; Xie *et al.*, 2008; Campos-Cantón *et al.*, 2008, 2010).

It is known that with piecewise linear functions one can achieve the generation of multiscroll chaotic attractors, which are based on the location of the equilibrium points introduced to the system along with the commutation law or threshold that bounds the scrolls and gives a specific direction to the flow. The multiple papers about this topic have been presented as different ideas and several theories have been developed to explain how to generate multi-scroll chaotic attractors. A natural question is the following: is there a theory that explains all these approaches as one? For example, Yalçin *et al.* (2002), reported that a 1D, 2D and 3D-grid of scrolls may be introduced locating them around the equilibrium points in space using a step function. Lü *et al.* (2004 A); Deng & Lü (2007) presented an approach using hysteresis that enables the creation of 1D n-scrolls, 2D $n \times m$-grid scrolls and 3D $n \times m \times l$-grid scrolls chaotic attractors.

In this work, we present a generalized theory that is capable of explaining different approaches as saturation, threshold and step functions in $\Re^3$. This class of systems is constructed with *unstable dissipative systems* (UDS) (Campos-Cantón *et al.*, 2010) and a control law to display various multi-scroll strange attractors. The multi-scroll strange attractors result from the combination of several unstable "one-spiral" trajectories by means of a switching given by the control law. Without loss of generality we focus our study to the simple jerk equation and a switching control law to generate PWL systems that produce multiscroll attractors.

This paper is organized as follow: In Section 2, we introduce the UDS theory to explain the generation of multi-scrolls attractors, along with some examples using the jerky equation. In Section 3 we use the UDS theory to generate different approaches as step function, saturation and hysteresis. In Section 4 we present a mechanism for Brownian motion generation in terms of UDS, and in Section 5 we draw conclusions.

2. SWITCHED SYSTEMS BASED ON UNSTABLE DISSIPATIVE SYSTEMS

We consider the class of affine linear system given by

$$\dot{\chi} = A\chi + B, \tag{1}$$

where $\chi = [x_1, \ldots, x_n]^T \in \Re^n$ is the state variable, $B = [\beta_1, \ldots, \beta_n]^T \in \Re^n$ stands for a real vector, $A = [\alpha_{ij}] \in \Re^{n\times n}$ denotes a linear operator. Considering that matrix A is not singular then the equilibrium point is located at $\chi^* = -A^{-1}B$. The dynamics of the system is given by matrix A due to define a vector field Ax. Suppose

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that the matrix A has j negative eigenvalues $\lambda_1, \ldots, \lambda_j$ and $n-j$ positive eigenvalues $\lambda_{j+1}, \ldots, \lambda_n$. Let $\{v_1, \ldots, v_n\}$ be the corresponding set of eigenvectors. Then the stable and unstable subspaces of the affine linear system (1), E^s and E^u, are the linear subspaces spanned by $\{v_1, \ldots, v_j\}$ and $\{v_{j+1}, \ldots, v_n\}$, respectively; i.e.,

$$E^s = Span\{v_1, \ldots, v_j\},$$
$$E^u = Span\{v_{j+1}, \ldots, vn\}.$$

According to the above discussion and considering real and complex eigenvalues, it is possible to define a UDS as follows:

Definition 1. A system given by (1) in $\Re^n$ and eigenvalues λ_i, with $i = 1, \ldots, n$. We said that system (1) is a UDS if $\sum_{i=1}^{n} \lambda_i < 0$, and at least one λ_i is a positive real eigenvalue or two λ_i are complex eigenvalues with positive real part $Re\{\lambda_i\} > 0$. None of them is pure imaginary eigenvalue.

The next proposition is important to mention in order to realize what kinds of behaviors are possible to find in the system given by (1).

Proposition 2. Let the system (1) be a UDS with ordered real and complex eigenvalues set $\Lambda = \{\lambda_1, \ldots, \lambda_n\}$ and $Re\{\lambda_1\} \leq \ldots \leq Re\{\lambda_j\} < 0 < Re\{\lambda_{j+1}\} \leq \ldots \leq Re\{\lambda_n\}$. Then, the system has a stable manifold $E^s \subset \Re^n$ and another unstable $E^u \subset \Re^n$ with $1 \leq j \leq n$ and the following statements are true:
(a) All initial condition $\chi_0 \in \Re^n / E^s$ leads to an unstable trajectory that goes to infinity.
(b) All initial condition $\chi_0 \in E^s$ leads to a stable trajectory that settles down at χ^* and the system does not generate oscillations.
(c) The basin of attraction $\mathcal{B}$ is $E^s \subset \Re^n$.

Now, we consider a switching system based on the affine linear system (1) given by

$$\dot{\chi} = A\chi + B(\chi),$$

$$B(\chi) = \begin{cases} B_1, & \text{if } \chi \in D_1; \\ \vdots & \vdots \\ B_k, & \text{if } \chi \in D_k. \end{cases} \quad (2)$$

Where $\Re^n = \cup_{i=1}^{k} D_i$ and $\cap_{i=1}^{k} D_i = \emptyset$. Thus, the equilibria of the system (2) is $\chi_i^* = -A^{-1}B_i$, with $i = 1, \ldots, k$. So the goal is to define vectors B_i which can generate a class of dynamical systems in $\mathbf{R}^n$ with oscillations into an attractor, that is, the flow $\Phi(\chi(0))$ of the system (2) is trapped in an attractor $\mathcal{A}$ by defining at least two vectors B_1 and B_2. This class of systems can display various multi-scroll strange attractors as a result of the combination of several unstable "one-spiral" trajectories by means of the commutation of $B(\chi)$, i.e., we are interested in a vector field which can yield multi-scroll attractors constitute by a commuted vector, B_i with $i = 1, \ldots, k$ and $k \geq 2$. Each domain, $\mathcal{D}_i \subset \Re^n$, contains the equilibrium $\chi_i^* = -A^{-1}B_i$. According to the above discussion we can define a multi-scroll chaotic system based on UDS as follows:

Definition 3. A system given by (2) in $\Re^n$ and equilibrium points χ_i^*, with $i = 1, \ldots, k$ and $k > 2$. We said that system (2) is a multi-scroll chaotic system if each χ_i^* contains oscillations around and the flow $\phi(\chi_0)$ generates an attractor $\mathcal{A} \subset \Re^n$.

According to the above discussion, it is possible to define two types of UDS in $\Re^3$, and two types of corresponding equilibria, for more details see Campos-Cantón *et al.* (2010, 2012).

Definition 4. Consider the system (1) in $\Re^3$ with eigenvalues λ_i, $i = 1, 2, 3$ such that $\sum_{i=1}^{3} \lambda_i < 0$. Then the system is said to be an UDS of *type I* (UDS-I) if one of its eigenvalues is negative real and the other two are complex conjugate with positive real part; and it is said to be of *type II* (UDS-II) if one of its eigenvalues is positive real and the other two are complex conjugate with negative real part.

In order to illustrate our approach we consider the particular case of the linear ordinary differential equation(ODE) written in the jerky form as $d^3x/dt^3 + \alpha_{33}d^2x/dt^2 + \alpha_{32}dx/dt + \alpha_{31}x + \beta_3 = 0$, representing the state space equations of (1), where the matrix A and the vector B are described as follows:

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -\alpha_{31} & -\alpha_{32} & -\alpha_{33} \end{pmatrix}; B = \begin{pmatrix} 0 \\ 0 \\ \beta_3 \end{pmatrix}, \quad (3)$$

where the coefficients $\alpha_{31}, \alpha_{32}, \alpha_{33}, \beta_3 \in \Re$ may be any arbitrary scalars that satisfy the definition 4. The characteristic polynomial of matrix A given by (3) takes the following form:

$$\lambda^3 + \alpha_{33}\lambda^2 + \alpha_{32}\lambda + \alpha_{31}. \quad (4)$$

For simplicity, we vary the coefficient $\alpha_{31} \in \Re$ and set the others coefficients at $\alpha_{32} = 1$, $\alpha_{33} = 1$. The coefficient α_{31} has to assure the system will be UDS-I or UDS-II. Fig. 1 shows the location of the roots, for example the UDS's-II are given for $\alpha_{31} < 0$, and the UDS's-I for $\alpha_{31} > 1$. The system has a sink for $0 < \alpha_{31} < 1$. We are setting $\alpha_{31} = 1.5$ in order to assure a UDS-I, with these values the eigenvalues result in $\lambda_1 = -1.20$, $\lambda_{2,3} = 0.10 \pm 1.11i$, which satisfy Definition 4 for UDS-I. The parameter β_3 is governed by the following switching control law (SCL):

$$\beta_3 = \begin{cases} 0.9, & \text{if } x_1 \geq 0.3; \\ 0 & \text{otherwise.} \end{cases} \quad (5)$$

The equilibrium points of the system (2) using the matrix A and vector B defined in (3) and the SCL (5) are $\chi_1^* = (0.6, 0, 0)^T$ with $B_1 = (0, 0, 0.9)^T$ and χ_2^* at the origin with $B_2 = (0, 0, 0)^T$.

Figure 2 a) depicts the projection of the double-scroll attractor onto the (x_1, x_2) plane generated by the β_3 SCL (5) under equations (2)-(3).

Now, if we change the control signal given by SCL then it is possible to generate an attractor with triple-scroll. Therefore the β_3 parameter is given as follows:

$$\beta_3 = \begin{cases} 0.9, & \text{if } 0.3 \leq x_1; \\ 0 & \text{if } -0.3 < x_1 < 0.3; \\ -0.9, & \text{if } x_1 \leq -0.3. \end{cases} \quad (6)$$

Notice that $\chi_3^* = -\chi_1^*$. This issue is intentionally defined to illustrate the symmetry scrolls. Figure 2 b) shows the