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Research article

Adaptive output-feedback control for switched stochastic uncertain nonlinear systems with time-varying delay

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ABSTRACT

This paper addresses the problem of adaptive output-feedback control for a class of switched stochastic time-delay nonlinear systems with uncertain output function, where both the control coefficients and time-varying delay are unknown. The drift and diffusion terms are subject to unknown homogeneous growth condition. By virtue of adding a power integrator technique, an adaptive output-feedback controller is designed to render that the closed-loop system is bounded in probability, and the state of switched stochastic nonlinear system can be globally regulated to the origin almost surely. A numerical example is provided to demonstrate the validity of the proposed control method.

1. Introduction

In past years, switched nonlinear systems have received considerable attention for their wide utilization in many branches of science and industry, see Refs. [1–6] and the references therein. A main problem of switched systems is global asymptotical stability under arbitrary switchings. In Ref. [1], it can be indicated that the existence of a common Lyapunov function for all subsystems is a necessary and sufficient condition for a switched system to be asymptotically stable under arbitrary switchings. But a common Lyapunov function may be very difficult to find, and thus focus is transformed to switched nonlinear systems with special structures [4,7]. As a special case of switched systems, switched stochastic nonlinear systems play an important role in physical and engineering systems with stochastic disturbances, such as [8,9]. The stability theory of switched stochastic systems was proposed in Refs. [10,11], and the problem of stabilizer design was further investigated in Refs. [12,13]. Subsequently, [14] addressed the problem of global stabilization for switched stochastic nonlinear systems in strict-feedback form under arbitrary switchings.

However, the aforementioned works only studied global state-feedback stabilization issues for switched stochastic nonlinear systems. Since only the system output can be measured in many real applications, the output-feedback control is one of the most important problems of switched stochastic nonlinear systems. As a matter of fact, the global output-feedback control problem for such systems has been limitedly studied in recent years. Ref. [15] addressed the output-feedback stabilization issue for networked control systems with random delays modeled by Markov chains. Ref. [16] investigated the problem of anti-windup design for stochastic Markovian switching systems with saturation nonlinearity. The problem of output-feedback stabilization for switched stochastic nonlinear systems under arbitrary switchings was studied in Ref. [17]. By a modified average dwell-time method, an adaptive output-feedback controller for switched nonlinear stochastic systems was designed in Ref. [18].

It is well-known that time-delay phenomena are usually encountered in many practical control systems, such as ecological systems, industrial processes, telerobotic systems, earth controlled satellite devices, see Refs. [19–25] and the references therein. Time delay may become the sources of instability and degrade the performance of systems. Ref. [26] studied the problem of global asymptotic stability of fractional-order BAM neural networks, where both time-delay and impulsive effects were considered. Ref. [27] investigated adaptive neural control problem of stochastic nonlinear time-delay systems with multiple constraints. For switched stochastic nonlinear systems with time delays, in Ref. [28], an output-feedback controller independent of switching signals and time delays was constructed by the backstepping method. Subsequently, an adaptive fuzzy output-feedback tracking controller was designed for switched

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stochastic nonlinear systems with time delays in Ref. [29]. Spontaneously, an interesting question can be proposed: Is it possible to find suitable restrictions on nonlinearities and output function, and how can we design a global adaptive output-feedback controller for switched stochastic uncertain nonlinear system with time-varying delay? Inspired by Refs. [30,31], the problem of adaptive output-feedback control for switched stochastic uncertain time-delay nonlinear systems can be solved by adding a power integrator technique. The main contributions of this paper are summarized as: (i) compared with output $y = z_1$ in Refs. [25,28], the state z_1 cannot be precisely estimated by output y owing to uncertain output function $y = h(z_1)$, a novel full-order observer is proposed. Particularly, $y = \beta z_1$ in Ref. [32] is a special case of system (1) with the control coefficients $b_1 = b_2 = \cdots b_n = 1$. (ii) distinct from the situation that the nonlinearities satisfy unknown linear growth condition in Ref. [33], a new adaptive output-feedback controller is designed under a weak situation that the nonlinearities satisfy unknown homogeneous growth condition. (iii) our results extend the existing global stabilization results for non-switched stochastic systems without time delay to switched stochastic time-delay nonlinear systems.

The rest of this paper is organised as follows. Section 2 reviews the Lyapunov stability theory of stochastic time-delay nonlinear system. The main result is presented in Section 3, where observer and adaptive controller are explicitly designed. A numerical example is established in Section 4. Concluding remarks are included in Section 5. The proofs of four propositions are given in Appendix.

2. Problem formulation and preliminaries

In this paper, we consider a class of switched stochastic time-delay nonlinear systems described by

$$\begin{aligned} dz_i(t) &= (b_i z_{i+1}(t) + \psi_{i\sigma(t)}(t, y(t), y(t-d(t)), \theta))dt \\ &\quad + \phi^T_{i\sigma(t)}(t, y(t), y(t-d(t)), \theta)d\omega, \\ dz_n(t) &= (b_n u(t) + \psi_{n\sigma(t)}(t, y(t), y(t-d(t)), \theta))dt \\ &\quad + \phi^T_{n\sigma(t)}(t, y(t), y(t-d(t)), \theta)d\omega, \\ y(t) &= h(z_1(t)), \quad i = 1, \ldots, n-1 \end{aligned} \tag{1}$$

where $z = (z_1, \ldots, z_n)^T \in \mathbb{R}^n$, $u(t) \in \mathbb{R}$ and $y(t) \in \mathbb{R}$ are the system state, input and output, respectively. The control coefficients $b_i > 0$, $i = 1, \ldots, n$, are unknown constants. $d(t) : \mathbb{R}^+ \to [0, d]$, is unknown time-varying delay satisfying $\dot{d}(t) \le \gamma < 1$ for a known constant γ. The initial value $\{z(s) : -d \le s \le 0\} = \xi \in C^b_{\mathcal{F}0}([-d, 0]; \mathbb{R}^n)$, where $C^b_{\mathcal{F}0}([-d, 0]; \mathbb{R}^n)$ expresses the family of all $\mathcal{F}_0$-measurable bounded $C([-d, 0]; \mathbb{R}^n)$-valued random variables $\xi = \{\xi(s): -d \le s \le 0\}$. ω is an r-dimensional standard Wiener process defined on a probability space $(\omega, \mathcal{F}, \mathcal{F}_t, P)$ with ω being a sample space, $\mathcal{F}$ being a σ-field, $\mathcal{F}_t$ being a filtration and P being a probability measure. $\sigma(t)$ is the switching signal taking its values in a finite set $M = \{1, \ldots, m\}$ and m is the number of subsystems. $\theta \in \mathbb{R}^\iota$ is a vector of uncertain parameters with integer $\iota > 0$. For $i = 1, \ldots, n$ and $k \in M$, the drift terms $\psi_{ik} : \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}$ and the diffusion terms $\phi_{ik} : \mathbb{R}^+ \times \mathbb{R} \times \mathbb{R} \to \mathbb{R}^r$, are locally Lipschitz continuous with their arguments and satisfy $\psi_{ik}(t, 0, 0, \theta) = 0$, $\phi_{ik}(t, 0, 0, \theta) = 0$. $h : \mathbb{R} \to \mathbb{R}$ is an uncertain C^2 function satisfying $h(0) = 0$, where C^2 denotes the set of all functions with continuous second partial derivatives. In addition, we assume that the state of system (1) does not jump at the switching instants, i.e., the trajectory $z(t)$ is everywhere continuous. For simplicity, the time variable t in the delay-free states is omitted, and the time variable t in the delay states is described as $x(t - d(t)) = x_d$ in the rest of this paper.

Next, Some preliminary results for the following stochastic time-delay system are presented.

$$dx = f(t, x, x_d)dt + g^T(t, x, x_d)d\omega, \forall t \ge 0 \tag{2}$$

with initial value $\{x(s) : -d \le s \le 0\} = \xi \in C^b_{\mathcal{F}0}([-d, 0]; \mathbb{R}^n)$, where $d(t) : \mathbb{R}^+ \to [0, d]$ is a time-varying delay, and ω is an r-dimensional standard Wiener process defined on a probability space $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\ge0}, P)$. $f : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ and $g^T : \mathbb{R}^+ \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz continuous with $f(t, 0, 0) = 0$ and $g(t, 0, 0) = 0$.

Definition 2.1. [34] For any given $V(t, x) \in C^{1,2}$ associated with system (2), the infinitesimal generator $\mathscr{L}$ is defined as $\mathscr{L}V = \frac{\partial V}{\partial t} + \frac{\partial V}{\partial x}f + \frac{1}{2}Tr\{g\frac{\partial^2 V}{\partial x^2}g^T\}$ with $\frac{1}{2}Tr\{g\frac{\partial^2 V}{\partial x^2}g^T\}$ being called as the Hessian term of $\mathscr{L}$.

Definition 2.2. [34] The equilibrium $x(t) = 0$ of the stochastic time-delay system (2) is said to be globally asymptotically stable in probability if for any $\varepsilon > 0$, there is a class $\mathcal{KL}$ function $\beta(\cdot, \cdot)$ such that $P\{|x| \le \beta(\|\xi\|, t)\} \ge 1 - \varepsilon$ for any $t \ge 0$, $\xi \in C^b_{\mathcal{F}0}([-d, 0]; \mathbb{R}^n)$, where $\|\xi\| = \sup_{\theta\in[-d, 0]}|x(\theta)|$.

Lemma 2.1. [25] For system (2), if there is a function $V(t, x) \in C^{1,2}$, two class $\mathcal{K}_\infty$ functions α_1, α_2, and a class $\mathcal{K}$ function α_3 such that

$$\alpha_1(|x|) \le V(t, x) \le \alpha_2(\sup_{-d\le s\le 0} |x(t+s)|) \quad \text{and} \quad \mathscr{L}V(t, x) \le -\alpha_3(|x|) \tag{3}$$

then there exists a unique solution on $[-d, \infty)$ for (2), and the equilibrium $x(t) = 0$ is globally asymptotically stable in probability and $P\{\lim_{t\to\infty}|x| = 0\} = 1$.

Lemma 2.2. [36] Suppose $V : \mathbb{R}^+ \times \mathbb{R}^n \to \mathbb{R}$ is a homogeneous function of degree τ with respect to the dilation weight Δ. Then the following results hold:

(i) $\partial V/\partial x_i$ is homogeneous of degree $\tau - r_i$ with r_i being the homogeneous weight of x_i.
(ii) There is a positive constant c such that $V(t, x) \le c|x|^\tau_\Delta$. Moreover, if $V(t, x)$ is positive definite, Then $\underline{c}|x|^\tau_\Delta \le V(t, x)$ for a constant $\underline{c} > 0$.

Lemma 2.3. For $x \in \mathbb{R}$, $y \in \mathbb{R}$, and $p \ge 1$, the following inequalities hold:

$$|x + y|^p \le 2^{p-1}|x^p + y^p|,$$

$$(|x| + |y|)^{\frac{1}{p}} \le |x|^{\frac{1}{p}} + |y|^{\frac{1}{p}} \le 2^{\frac{p-1}{p}}(|x| + |y|)^{\frac{1}{p}}.$$

If $p \ge 1$ is a ratio of two odd integers,

$$|x - y|^p \le 2^{p-1}|x^p - y^p|, \qquad |x^{\frac{1}{p}} - y^{\frac{1}{p}}| \le 2^{1-\frac{1}{p}}|x - y|^{\frac{1}{p}}.$$

Lemma 2.4. For any positive real numbers c, d and any real-valued function $\gamma(x, y) > 0$, the following inequality holds:

$$|x|^c|y|^d \le \frac{c}{c+d}\gamma(x, y)|x|^{c+d} + \frac{d}{c+d}\gamma^{-\frac{c}{d}}(x, y)|y|^{c+d}.$$

3. Main results

In this section, we aim to design an adaptive output-feedback controller such that all signals of the closed-loop system are bounded

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