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#### Research article

# Mixed $H_{\infty}$ and passive filtering for switched Takagi-Sugeno fuzzy systems with average dwell time

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#### ABSTRACT

This paper investigates the mixed  $H_\infty$  and passive filtering problem for switched Takagi-Sugeno (T-S) fuzzy systems with average dwell time (ADT) in both continuous-time and discrete-time contexts. To deal with this problem, a new performance index is proposed for switched systems. This new performance index can be viewed as the mixed weighted  $H_\infty$  and passivity performance index. Based on this new performance index, the weighted  $H_\infty$  filtering problem and the passive filtering problem for switched T-S fuzzy systems can be solved in a unified framework. Combining the multiple Lyapunov functions approach with a matrix decoupling technique, new sufficient conditions for the existence of mixed weighted  $H_\infty$  and passive filters are obtained for switched T-S fuzzy systems. All these conditions are expressed in terms of linear matrix inequalities (LMIs). The desired filters can be constructed by solving these LMIs. Finally, numerical examples and practical examples are provided.

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#### 1. Introduction

During the past decades, the switched systems attracted more and more attention because of many physical or man-made systems possessing switching features [1]. A typical switched system is composed of a finite number of continuous-time or discrete-time subsystems and a switching signal which orchestrates the switching between these subsystems. Up to now, two stability issues have been well addressed in the literature, i.e., the stability under arbitrary switching and the stability under constrained switching. As one typical example of the constrained switching, the average dwell time (ADT) logic is proposed in Ref. [2]. By using the multiple Lyapunov functions approach [3], stability analysis and controller synthesis problems are addressed for switched systems with ADT switching [4–17]. The dwell time switching can be covered by the ADT switching [1], and the arbitrary switching can be viewed as an extreme case of the ADT switching [4]. Therefore, it is important and theoretically significant for us to study the switched systems with ADT.

As we know, the T-S fuzzy model is proven to be an effective tool in approximating most complex nonlinear systems [18], which utilizes local linear system description for each rule. The last sev-

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eral decades have witnessed the more and more applications of T-S fuzzy model in the analysis and synthesis of dynamical systems [19–27]. Recently, the T-S fuzzy model has been extended to describe each nonlinear subsystem of switched nonlinear systems. Many significant results have been obtained for switched T-S fuzzy systems [28–30].

Due to the fact that the state variables of a dynamic system are not always accessible for direct measurement, it is practical and important for us to estimate the state of such system through available measurements. Among the various methods of the state estimation, the  $H_{\infty}$  filtering keeps attracting more and more attention because of without consideration of the statistical properties of disturbances. Recently, a lot of efforts have been made to investigate the  $H_{\infty}$  filtering problem for T-S fuzzy systems [31–33]. However, less attention has been paid to the  $H_{\infty}$  filtering problem for switched T-S fuzzy systems. The  $H_{\infty}$  filtering problem for the discrete-time switched T-S fuzzy system under both fast and slow switching was studied in Ref. [34]. On the other hand, the passivity theory was an effective tool to analyze and synthesize the dynamical systems [35,36]. Furthermore, the passivity concept was extended to the switched systems [37,38]. Then, a natural problem arises: Can the  $H_{\infty}$  filtering problem and the passive filtering problem of switched systems be solved in a unified framework?

Very recently, some work have studied the mixed  $H_{\infty}$  and passive filtering problem for singular systems [39], networked Markov jump systems [40] and discrete fuzzy neural networks [41]. However, to

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the best of our knowledge, there is no attention paid to the mixed  $H_{\infty}$  and passive control problem for switched systems. Different from the above work, the mixed  $H_{\infty}$  and passive filtering problem of switched systems will be another case. On one hand, it has been pointed out that the switched systems can only achieve a weighted  $H_{\infty}$  performance index [42]; On the other hand, the existence of switching between the subsystems makes the mixed  $H_{\infty}$  and passive filtering problem of switched systems much more complicated. Therefore, it is important and necessary for us to study the mixed  $H_{\infty}$  and passive filtering problem of switched systems.

The main contributions of this paper are listed as follows: I) A new performance index is proposed for switched systems. This new performance index covers the weighted  $H_{\infty}$  performance index and the passivity performance index as special cases. II) The weighted  $H_{\infty}$  filtering problem and the passive filtering problem for switched T-S fuzzy systems are solved in a unified framework. III) New sufficient conditions are derived to guarantee the filtering error system to be globally uniformly asymptotically stable with a prescribed mixed weighted  $H_{\infty}$  and passivity performance index.

The rest of this paper is organized as follows. Preliminaries and problem formulation are presented in Section 2. The main results are given in Section 3. In Section 4, numerical examples and practical examples are presented. Finally, some conclusions are included in Section 5.

Notations: The notations used in this paper are fairly standard. For a matrix M,  $He\{M\}$  denotes  $M + M^T$ . The symbol "\*" in a matrix stands for the transposed elements in the symmetric positions. The superscript "T" is the matrix transposition. N denotes the set of the natural numbers.  $R^n$  denotes the *n*-dimensional Euclidean space. I and 0 represent the identity matrix and the zero matrix respectively in the block matrix with appropriate dimensions. The notation  $\|\cdot\|$  refers to the Euclidean vector norm.  $L_2[0,\infty)$  is the space of square-integrable. For  $v(t) \in L_2[0, \infty)$ , its norm is given by  $||v(t)||_2 =$  $\sqrt{\int_0^\infty v(t)^T v(t) dt}$ .  $l_2[0,\infty)$  is the space of square summable infinite sequence. For  $v(t) \in l_2[0, \infty)$ , its norm is given by  $||v(t)||_2 =$  $\sqrt{\sum_{t=0}^{\infty} |v(t)|^2}$ .  $C^1$  denotes the space of continuously differentiable function. The  $P > 0 \ (\ge 0, < 0, \le 0)$  are used to denote a positive definite (semi-positive definite, negative definite, semi-negative definite) matrix P. If not explicitly stated, matrices are assumed to have compatible dimensions.

#### 2. Preliminaries and problem formulation

In this paper, let us consider the switched T-S fuzzy system with every subsystem described by the following T-S fuzzy model with r plant rules.

Rule m for a subsystem  $\sigma(t)$ : IF  $v_{\sigma(t)1}(t)$  is  $M_{\sigma(t)1}$  m and  $\cdots$  and  $v_{\sigma(t)p}(t)$  is  $M_{\sigma(t)pm}$ ,

**THEN** 

$$\begin{cases} \delta x(t) = A_{\sigma(t)m} x(t) + B_{\sigma(t)m} w(t), \\ y(t) = C_{\sigma(t)m} x(t) + D_{\sigma(t)m} w(t), \\ z(t) = E_{\sigma(t)m} x(t), \end{cases}$$
(1)

where

$$\delta x(t) = \begin{cases} \dot{x}(t), & \text{for continuous-time switched systems,} \\ x(t+1), & \text{for discrete-time switched systems,} \end{cases}$$

 $x(t) \in \mathbb{R}^n$  is the state vector,  $y(t) \in \mathbb{R}^m$  is the measurement vector,  $z(t) \in \mathbb{R}^p$  is the output signal to be estimated, and  $w(t) \in \mathbb{R}^l$  is the disturbance input that belongs to  $L_2[0,\infty)$  for continuous-time switched systems and  $l_2[0,\infty)$  for discrete-time switched systems. The switching signal  $\sigma(t):[0,+\infty)\to S=\{1,2,\ldots,N\}$  is a piecewise constant function of time, where N is the number of subsystems.

For a switching sequence  $0=t_0 < t_1 < \cdots < t_k < t_{k+1} < \cdots$ ,  $\sigma(t)$  is continuous from left everywhere. When  $t \in [t_k, t_{k+1})$ , the  $\sigma(t_k)=i$  subsystem is activated. For  $\sigma(t)=i$ ,  $A_{im}$ ,  $B_{im}$ ,  $C_{im}$ ,  $D_{im}$  and  $E_{im}$  are constant real matrices with appropriate dimensions of the mth local model of the subsystem.  $v_i(t)=(v_{i1}(t),v_{i2}(t),\ldots,v_{ip}(t))$  are some measurable premise variables, and  $M_{ilm}(l=1,2,\ldots,p)$  are fuzzy sets.

By fuzzy blending, the final output of the ith subsystem is inferred as follows

$$\begin{cases} \delta x(t) = \sum_{m=1}^{r} h_{im}(t) \left[ A_{im} x(t) + B_{im} w(t) \right], \\ y(t) = \sum_{m=1}^{r} h_{im}(t) \left[ C_{im} x(t) + D_{im} w(t) \right], \\ z(t) = \sum_{m=1}^{r} h_{im}(t) E_{im} x(t), \end{cases}$$
(2)

where  $h_{im}(t) = v_{im}(t) / \sum_{m=1}^r v_{im}(t), v_{im}(t) = \prod_{l=1}^p M_{ilm}(v_{il}(t)), r$  is the number of IF-THEN rules, and  $M_{ilm}(v_{il}(t))$  is the grade of the membership function of  $v_{il}$  in  $M_{ilm}$ . For all t and  $m=1,2,\ldots,r$ , it is assumed that  $v_{im}(t) \geq 0$ . It is obvious that the normalized membership function  $h_{im}(t)$  satisfies

$$h_{im}(t) \ge 0, \quad \sum_{m=1}^{r} h_{im}(t) = 1.$$
 (3)

In this paper, the filter to be designed is assumed to have the following form

Rule m: IF  $v_{i1}(t)$  is  $M_{i1m}$  and  $\cdots$  and  $v_{ig}(t)$  is  $M_{igm}$ , THEN

$$\begin{cases} \delta x_f(t) = A_{fim} x_f(t) + B_{fim} y(t), \\ z_f(t) = E_{fim} x_f(t), \end{cases}$$
(4)

where  $x_f(t) \in \mathbb{R}^n$  is the filter state vector,  $z_f(t) \in \mathbb{R}^p$  is the output signal of the filter, and the matrices  $A_{fim}$ ,  $B_{fim}$ ,  $E_{fim}$  are the filter parameters to be designed. The final output of (4) is inferred as follows

$$\begin{cases} \delta x_{f}(t) = \sum_{m=1}^{r} h_{im}(t) \left[ A_{fim} x_{f}(t) + B_{fim} y(t) \right], \\ z_{f}(t) = \sum_{m=1}^{r} h_{im}(t) E_{fim} x_{f}(t). \end{cases}$$
(5)

Combing (2) with (5) and defining  $\widetilde{x}(t) = [x^T(t), x_f^T(t)]^T$  and  $e(t) = z(t) - z_f(t)$ , the following filtering error subsystem can be obtained

$$\begin{cases} \delta \widetilde{x}(t) = \widehat{A}_{i}(t)\widetilde{x}(t) + \widehat{B}_{i}(t)w(t), \\ e(t) = \widehat{E}_{i}(t)\widetilde{x}(t), \end{cases}$$
 (6)

where

$$\begin{split} \widehat{A}_i(t) &= \begin{bmatrix} A_i(t) & 0 \\ B_{fi}(t)C_i(t) & A_{fi}(t) \end{bmatrix} = \sum_{m=1}^r \sum_{n=1}^r h_{im}(t)h_{in}(t) \begin{bmatrix} A_{im} & 0 \\ B_{fin}C_{im} & A_{fin} \end{bmatrix}, \\ \widehat{B}_i(t) &= \begin{bmatrix} B_i(t) \\ B_{fi}(t)D_i(t) \end{bmatrix} = \sum_{m=1}^r \sum_{n=1}^r h_{im}(t)h_{in}(t) \begin{bmatrix} B_{im} \\ B_{fin}D_{im} \end{bmatrix}, \\ \widehat{E}_i(t) &= \begin{bmatrix} E_i(t) & -E_{fi}(t) \end{bmatrix} = \sum_{m=1}^r \sum_{n=1}^r h_{im}(t)h_{in}(t) \begin{bmatrix} E_{im} & -E_{fin} \end{bmatrix}. \end{split}$$

To obtain the desired results, the following definitions are needed.

**Definition 1.** [43] A continuous function  $\alpha$ :  $[0, \infty) \to [0, \infty)$  is said to be of class  $\mathcal{K}$  if it is strictly increasing and  $\alpha(0) = 0$ . If  $\alpha$  is also unbounded, then it is said to be of class  $\mathcal{K}_{\infty}$ . A function  $\beta$ :  $[0, \infty) \times$ 

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