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Research article

An adaptive three-stage extended Kalman filter for nonlinear discrete-time system in presence of unknown inputs

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ABSTRACT

Considering the performances of conventional Kalman filter may seriously degrade when it suffers stochastic faults and unknown input, which is very common in engineering problems, a new type of adaptive three-stage extended Kalman filter (AThSEKF) is proposed to solve state and fault estimation in nonlinear discrete-time system under these conditions. The three-stage U–V transformation and adaptive forgetting factor are introduced for derivation, and by comparing with the adaptive augmented state extended Kalman filter, it is proven to be uniformly asymptotically stable. Furthermore, the adaptive three-stage extended Kalman filter is applied to a two-dimensional radar tracking scenario to illustrate the effect, and the performance is compared with that of conventional three stage extended Kalman filter (ThSEKF) and the adaptive two-stage extended Kalman filter (ATEKF). The results show that the adaptive three-stage extended Kalman filter is more effective than these two filters when facing the nonlinear discrete-time systems with information of unknown inputs not perfectly known.

1. Introduction

During the last decades, the problem of joint state and fault estimation for systems with unknown inputs has received intensive attentions [1–5]. To solve the problem, researchers have proposed numerous different approaches like unknown input observer [1,3,6,7], H_∞ filtering [8–10], two-stage Kalman filter [11–18], there-stage Kalman filter (ThSKF) [19,20], and et al. Klaman filtering is a widely used algorithm in many applications, such as robust control, adaptive filtering, parameter identification and estimation [21–31]. But since the conventional Kalman filter techniques can operate under the condition that accurate dynamic system model and statistical information of noise are perfectly known, the filter will diverge if it suffers stochastic faults and unknown inputs. The basic approach augmented state Kalman filter (ASKF) was proposed, which was to augment the state vector by adding additional components to represent the unknown inputs [11]. But the computational cost increased sharply with the augmented state dimension, and it becomes more serious when the dimension of fault and unknown inputs vector was comparable to that of the system state [12]. Friedland introduced a two-stage Kalman filter (TSKF) to reduce the computation cost and decoupled ASKF into two reduced-order filters, one of which was bias free filter that was dedicated to the state estimate, and the other one was bias filter that for the bias estimate. Then the optimum state estimate was obtained by coupling equations [11]. However, the filter was only optimal for constant bias. Alouani et al. extended the filter for random bias, and it was suboptimal only if an algebraic constraint on the correlation between the process noises of state and bias was satisfied, but the constraint was difficult to achieve [12]. To remove the algebraic constraint, an optimal two-stage Kalman filter (OTKF) was proposed by using two-stage U–V transformation [13]. Hsieh et al. has proven that OTKF was equivalent to ASKF and the two-stage estimators had the computational advantage over ASKF. And they demonstrated that the computational saving of the two-stage structure was because of the system-order reduction. However, OTKF was limited to the systems with unknown inputs that having a prescribed statistical model. For the case that the information of the random bias is unknown or incorrect, Kim et al. developed the adaptive TKF (ATKF) and analyzed the stability of the filter [14,32]. Based on the TKF, Hmida et al. proposed an optimal three-stage Kalman filter (OThSKF) for the state and fault estimation of linear systems with unknown inputs, which was derived by the application of a three-stage U–V transformation to decouple the ASKF covariance matrices [19]. ThSKF was also proved to be equivalent to ASKF, and the computational advantage of OThSKF

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was also verified. However, the filter was optimal unless the noise statistical properties of the fault and the unknown input were accurately known. In general case, the information of unknown inputs and fault are not perfectly known, a proportional integral ThSKF was developed by introducing the integral action technique [20]. And Xiao et al. proposed an adaptive ThSKF (AThSKF) by using adaptive forgetting factor technique, proved that AThSKF was equivalent to ASKF and analyzed the stability [33]. However the above filters are only available for linear discrete-time systems. This paper considers state and fault estimations for nonlinear systems with unknown inputs, and the information of fault and unknown inputs is not perfectly known.

Researchers extended TSKF to nonlinear system by applying an extended Kalman filter (EKF)-like nonlinear version of TKF (TEKF) [15–18]. EKF was the most widely used algorithm for the state estimation of the nonlinear systems [17,18]. In this filter, nonlinear filtering problems are converted into linear ones by using the first order approximation of Taylor series expansion, and then Kalman optimal estimation theory was utilized to estimate the state. However, the unknown inputs of nonlinear system may degrade the performance of TEKF because the prior information of the unknown inputs was partially known in practical situations. Subsequently an adaptive TEKF (ATEKF) was proposed for nonlinear stochastic systems with unknown constant or random inputs by adopting the forgetting factor technique, which was applied for the INS-GPS loosely coupled navigation system with unknown inputs successfully [16,18]. ATEKF was equivalent to the adaptive augmented state extended Kalman filter (ASEKF) and its stability was also analyzed [34]. In this paper, we propose an adaptive three-stage extended Kalman filter (AThSEKF) for nonlinear systems in presence of fault and unknown inputs of which the information is not perfectly known, and the stability of the filter is also analyzed.

The paper is organized as follows. Section 2, presents the derivation of the adaptive three-stage extended Kalman filter briefly. Section 3 discusses the stability of the proposed adaptive three-stage extended Kalman filter. Section 4 gives the comparison of the performance between adaptive two-stage extended Kalman filter, three-stage extended Kalman filter and adaptive three-stage extended Kalman filter. Finally, Section 5 gives the conclusion.

2. Adaptive three-stage extended Kalman filter

The nonlinear discrete-time stochastic systems with faults and unknown inputs can be described as

$$\mathbf{x}_{k+1} = \mathbf{f}_k(\mathbf{x}_k, \mathbf{b}_k, \mathbf{d}_k) + \mathbf{w}_k^x \tag{1a}$$

$$\mathbf{y}_k = \mathbf{h}_k(\mathbf{x}_k, \mathbf{b}_k, \mathbf{d}_k) + \mathbf{v}_k \tag{1b}$$

where $f(\cdot)$ and $h(\cdot)$ are assumed to be nonlinear continuously differentiable functions, $\mathbf{x}_{k+1} \in \Re^n$ is the state vector, $y_k \in \Re^m$ is the observation vector, $b_k \in \Re^p$ is the additive fault vector, $d_k \in \Re^q$ is unknown input vector.

The fault $\mathbf{b}_k$ and unknown input $\mathbf{d}_k$ are considered as stochastic processes with given wide-sense representation, thus the dynamics of the fault and unknown input can be given as [13,16,19].

$$\mathbf{b}_{k+1} = \mathbf{A}_k^b \mathbf{b}_k + \mathbf{w}_k^b \tag{1c}$$

$$\mathbf{d}_{k+1} = \mathbf{A}_k^d \mathbf{d}_k + \mathbf{w}_k^d \tag{1d}$$

where w_k^x, w_k^b, w_k^d and v_k denote the zero-mean uncorrelated Gaussian random noise sequences. The covariance matrix is

$$E\left[\begin{bmatrix} \mathbf{w}_k^x \\ \mathbf{w}_k^b \\ \mathbf{w}_k^d \\ \mathbf{v}_k \end{bmatrix} \begin{bmatrix} \mathbf{w}_j^x \\ \mathbf{w}_j^b \\ \mathbf{w}_j^d \\ \mathbf{v}_j \end{bmatrix}^T\right] = \begin{bmatrix} \mathbf{Q}_k^x & 0 & 0 & 0 \\ 0 & \mathbf{Q}_k^b & 0 & 0 \\ 0 & 0 & \mathbf{Q}_k^d & 0 \\ 0 & 0 & 0 & \mathbf{R}_k \end{bmatrix} \delta_{kj} \tag{1e}$$

where δ_{kj} is the Kronecker delta, $\delta_{kj} = 1$ if $k = j$, else $\delta_{kj} = 0$ if $k \neq j$. The initial state x_0, fault b_0 and unknown inputs d_0 are uncorrelated with the white noise sequences, which are assumed to satisfy the following equations

$$\begin{gathered} E(\mathbf{x}_0) = \widehat{\mathbf{x}}_0, E(\mathbf{b}_0) = \widehat{\mathbf{b}}_0, E(\mathbf{d}_0) = \widehat{\mathbf{d}}_0 \\ E\left[(\mathbf{x}_0 - \widehat{\mathbf{x}}_0)(\mathbf{x}_0 - \widehat{\mathbf{x}}_0)^T\right] = \mathbf{P}_0^x, E\left[\left(\mathbf{b}_0 - \widehat{\mathbf{b}}_0\right)\left(\mathbf{b}_0 - \widehat{\mathbf{b}}_0\right)^T\right] = \mathbf{P}_0^b, E\left[\left(\mathbf{d}_0 - \widehat{\mathbf{d}}_0\right)\left(\mathbf{d}_0 - \widehat{\mathbf{d}}_0\right)^T\right] = \mathbf{P}_0^d \\ E\left[(\mathbf{x}_0 - \widehat{\mathbf{x}}_0)\left(\mathbf{b}_0 - \widehat{\mathbf{b}}_0\right)^T\right] = \mathbf{P}_0^{xb}, E\left[(\mathbf{x}_0 - \widehat{\mathbf{x}}_0)\left(\mathbf{d}_0 - \widehat{\mathbf{d}}_0\right)^T\right] = \mathbf{P}_0^{xd}, E\left[\left(\mathbf{b}_0 - \widehat{\mathbf{b}}_0\right)\left(\mathbf{d}_0 - \widehat{\mathbf{d}}_0\right)^T\right] = \mathbf{P}_0^{bd} \end{gathered} \tag{1f}$$

The nonlinear functions $\mathbf{f}_k(\mathbf{x}_k, \mathbf{b}_k, \mathbf{d}_k)$ and $\mathbf{h}_k(\mathbf{x}_k, \mathbf{b}_k, \mathbf{d}_k)$ can be approximated as linear functions by Taylor series expansion as follows

$$\begin{aligned} \mathbf{f}_k(\mathbf{x}_k, \mathbf{b}_k, \mathbf{d}_k) &= \mathbf{f}_k\left(\widehat{\mathbf{x}}_k(+), \widehat{\mathbf{b}}_k(+), \overline{\mathbf{d}}_k(+)\right) + \mathbf{A}_k^x\left(\widehat{\mathbf{x}}_k(+), \widehat{\mathbf{b}}_k(+), \overline{\mathbf{d}}_k(+)\right)(\mathbf{x}_k - \widehat{\mathbf{x}}_k(+)) \\ &+ \mathbf{B}_k^x\left(\widehat{\mathbf{x}}_k(+), \widehat{\mathbf{b}}_k(+), \overline{\mathbf{d}}_k(+)\right)\left(\mathbf{b}_k - \widehat{\mathbf{b}}_k(+)\right) + \mathbf{E}_k^x\left(\widehat{\mathbf{x}}_k(+), \widehat{\mathbf{b}}_k(+), \overline{\mathbf{d}}_k(+)\right)\left(\mathbf{d}_k - \overline{\mathbf{d}}_k(+)\right) \\ &+ \boldsymbol{\varphi}_f\left(\mathbf{x}_k, \widehat{\mathbf{x}}_k(+), \widehat{\mathbf{b}}_k(+), \overline{\mathbf{d}}_k(+)\right) \end{aligned} \tag{2a}$$

$$\begin{aligned} \mathbf{h}_k(\mathbf{x}_k, \mathbf{b}_k, \mathbf{d}_k) &= \mathbf{h}_k\left(\widehat{\mathbf{x}}_k(-), \widehat{\mathbf{b}}_k(-), \overline{\mathbf{d}}_k(-)\right) + \mathbf{H}_k\left(\widehat{\mathbf{x}}_k(-), \widehat{\mathbf{b}}_k(-), \overline{\mathbf{d}}_k(-)\right)(\mathbf{x}_k - \widehat{\mathbf{x}}_k(-)) \\ &+ \mathbf{B}_k^y\left(\widehat{\mathbf{x}}_k(-), \widehat{\mathbf{b}}_k(-), \overline{\mathbf{d}}_k(-)\right)\left(\mathbf{b}_k - \widehat{\mathbf{b}}_k(-)\right) + \mathbf{E}_k^y\widehat{\mathbf{x}}_k(-), \widehat{\mathbf{b}}_k(-), \overline{\mathbf{d}}_k(-)\left(\mathbf{d}_k - \overline{\mathbf{d}}_k(-)\right) \\ &+ \boldsymbol{\varphi}_h\left(\mathbf{x}_k, \widehat{\mathbf{x}}_k(-), \widehat{\mathbf{b}}_k(-), \overline{\mathbf{d}}_k(-)\right) \end{aligned} \tag{2b}$$