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Research article

# Synchronization of heterogeneous linear networks with distinct inner coupling matrices

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ABSTRACT

In this paper, we study synchronization of heterogeneous linear networks with distinct inner coupling matrices. Firstly, for synchronous networks, we show that any synchronous trajectory will converge to a corresponding synchronous state. Then, we provide an invariant set, which can be exactly obtained by solving linear equations and then used for characterizing synchronous states. Afterwards, we use inner coupling matrices and node dynamics to successively decompose the original network into a new network, composed of the external part and the internal part. Moreover, this new network can be proved to synchronize to the above invariant set by constructing the corresponding desired Lyapunov-like functions for the internal part and the external part respectively. In particular, this result still holds if the coupling strength is disturbed slightly. Finally, examples with numerical simulations are given to illustrate the validity and applicability of our theoretical results.

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## 1. Introduction

A complex dynamical network is an abstract representation of some complex physical structure. Usually, it consists of some dynamical nodes connected by some edges in a complicate topology structure. In the past decades, complex dynamical networks have been used to model more and more real objects, such as biological oscillators [1,2], neural networks [3], power grids [4] and many other self-organizing systems [5,6]. Based on this fact, research on fundamental properties and dynamical behaviors of complex networks has become overwhelming [7–10].

As we all know, synchronization is a pretty common phenomenon in nature and man-made networks. It has attracted a lot of attention and triggered a huge relevant literature [11–17]. In complex dynamical networks, synchronization is mainly determined by three factors: the node dynamics, the outer coupling matrix and the inner coupling matrices. The node dynamics describes the behaviors of the dynamical nodes, the outer coupling matrix represents the topology structure while the inner coupling matrices represent the interaction among the nodes.

For the node dynamics, they are always assumed to be homogeneous for convenience, i.e., all nodes are identical and have the same dynamical behaviors. We call these networks as homogenous networks. Under this assumption, a lot of methods have been proposed to investigate synchronization. For example, a master stability function approach has been proposed in Ref. [18] for local synchronization; the Lyapunov method [19] and the connection graph stability method [20] are used to obtain synchronization criteria. However, nodes of many practical complex networks are nonidentical [21,22,25], e.g., a power grid may have different kinds of electric generators. These networks are called as heterogenous networks. It should be noted that many classical methods about homogeneous networks are not applicable for heterogeneous networks. Therefore, it is necessary to develop new methods for heterogeneous networks. Fortunately, there have amount of relevant results in recent years [23–26]. In Ref. [27], Wieland et al. proposed the internal model principle for synchronization of heterogeneous networks. Based on this method, several corresponding results have been obtained, e.g., Kim *et al.* [28] investigated the output consensus problem for heterogeneous uncertain linear multi-agent systems. While most researches focus on the existence of the internal model on a certain invariant set, [25] involves how to get an invariant set.

For the inner coupling matrices, many researchers pay attention to the case that they are identical [29,30]. But this condition is a little tighter and may fail to accurately capture certain features in some networks. For example, consider the interaction among a group of students, there are five different ways: picture friendship (i.e. two users sharing photos), Facebook friendship, dormmate, roommate and groupmate relationships [31]. Thus, it is significant to study

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synchronization of networks with distinct inner coupling matrices. However, since the distinction of inner coupling matrices, the diffusive condition of heterogeneous networks may no longer hold. To avoid this problem, one alternative way is to extend the concept of synchronization to a generalized form — output synchronization [32–34].

Different from output synchronization, we here just focus our attention on synchronization of heterogeneous linear networks with distinct inner coupling matrices. Specifically, we will introduce an invariant set to characterize the synchronous states and then propose our synchronization criterion based on this invariant set.

Firstly, for synchronous networks, we show that any synchronous trajectory will converge to the corresponding synchronous state. Especially, we find that there exist coupling-strength-dependent synchronous states in some of our networks (see Example 1). This is different from the homogenous networks, where all the synchronous states are independent to coupling strength. Then, in order to characterize the synchronous states that are independent to coupling strength, we introduce a linear invariant subspace by solving linear equations. Note that while [27] focuses on the existence of an invariant set, we here provide a general and practical way to obtain a linear invariant subspace.

Afterwards, based on distinct inner coupling matrices, we decompose the original network into two simpler parts: one has distinct inner coupling matrices and the other has the identical one. Moreover, by virtue of the above linear subspace, we further decompose the aforementioned part with identical inner coupling matrix into two sub-parts: one is the homogenous part and the other is the heterogeneous part. Thus, we get a new network with three parts: the first two parts can be seen as the external part, which can only synchronize to an equilibrium point of itself; the last part is the internal part, and if we restrict the three-part network on the subspace corresponding to this part, then it has identical node dynamics and identical inner coupling matrix. For this three-part network, a quadratic Lyapunov function, composed by two corresponding Lyapunov-like functions for the external part and the internal part respectively, is proposed to prove that this network with certain constraints can synchronize to the above invariant set. In particular, our synchronization criterion still holds if the coupling strength is disturbed slightly.

Note that some works [32] assumed that the synchronous states are known prior and then proposed corresponding control protocols to make the trajectories converge to these synchronous states. However, we here presuppose that the network has a linear distributed control protocol. Based on this supposition, we algebraically characterize the (unknown) synchronous states and propose a criterion for synchronizing to these synchronous states.

Finally, examples with numerical simulations are given to illustrate the validity and advantages of our theoretical results.

The paper is organized as follows. In Section 2, we present some basic definitions and notations. In Section 3, the characterization of synchronization states is presented. In Section 4, the criterion for synchronization is obtained. In Section 5, examples are presented with numerical simulations. The paper is concluded in Section 6.

## 2. Preliminaries

We consider the following heterogeneous linear network with distinct inner coupling matrices:

$$\dot{x}_i = A_i x_i - c\sum_{j=1}^{N} l_{ij}\Gamma_j x_j, \quad i = 1, \ldots, N, \tag{1}$$

where $x_i = (x_{i1}, \ldots, x_{in})^T \in \mathbb{R}^n$ is the state vector of the $i$-th node, $A_i$ is an $n \times n$ matrix, $c$ is the overall coupling strength, $\Gamma_j = \mathrm{diag}(\gamma_j^{(1)}, \ldots, \gamma_j^{(n)})$ $(j = 1, \ldots, N)$ is the distinct inner coupling matrix satisfying that for all $1 \le i \le n$, $\gamma_j^{(i)} > 0$, and $L = [l_{ij}] \in \mathbb{R}^{N\times N}$ is the outer coupling matrix with $l_{ii} = -\sum_{j=1, j\neq i}^{N} l_{ij}$ and $l_{ij} \ge 0$ for all $1 \le i{\neq}j \le N$. For network (1), we denote the solution $x(t)$ with the initial condition $x_0 = x(t_0)$ as $x(t, t_0, x_0)$.

Usually, the outer coupling matrix is required to be irreducible, which can be defined as follows.

**Definition 1.** [35] *A matrix $L \in \mathbb{R}^{N\times N}$ is reducible if there is a permutation matrix $P \in \mathbb{R}^{N\times N}$ such that*

$$P^T L P = \begin{pmatrix} B & C \\ 0_{n-r,r} & D \end{pmatrix}, \quad \text{where } 1 \le r \le n-1.$$

For the outer coupling matrix, the following general algebraic connectivity is an important index, which will be used in our synchronization criterion later.

**Definition 2.** [36] *For a Laplacian matrix $L$, its general algebraic connectivity is defined to be*

$$a_\xi(L) = \min_{x^T\xi=0, x\neq 0} \frac{x^T \widehat{L} x}{x^T \Psi x},$$

*where $\xi = (\xi_1, \ldots, \xi_N)^T$ with $\xi_i > 0$ $(i = 1, \ldots, N)$ and $\sum_{i=1}^{N} \xi_i = 1$, $\widehat{L} = (\Psi L + L^T \Psi)/2$, $\Psi = \mathrm{diag}(\xi_1, \ldots, \xi_N)$.*

In this paper, we assume that the outer coupling matrix $L$ is irreducible and $\xi$ is the left eigenvectors of $L$ corresponding to the zero eigenvalue. Thus, $a_\xi(L) > 0$. To study the synchronization of network (1) in this paper, we introduce the following definitions.

**Definition 3.** *A smooth function $s(t) : \mathbb{R} \to \mathbb{R}^n$ is said to be a synchronous state of network (1) if the function $S(t) = (s(t), \ldots, s(t))$ is a solution of network (1).*

**Definition 4.** *A solution $X(t) = (x_1(t), \ldots, x_N(t))$ of network (1) is said to be a synchronous trajectory if*

$$\lim_{t\to\infty} \left\| x_i(t) - x_j(t) \right\| = 0, \quad \forall 1 \le i, j \le N,$$

*where $\|\cdot\|$ denotes the Euclidean norm. Moreover, network (1) is said to achieve synchronization if all the solutions of network (1) are synchronous trajectories.*

Additionally, the following definitions will be used in Section 3.

**Definition 5.** [37] *Let $B$ be a subset of $\mathbb{R}^{Nn}$. $B$ is said to be invariant with respect to network (1) if for any $x_0 \in B$, $x(t, t_0, x_0) \in B$ for all $t \in \mathbb{R}$.*

**Definition 6.** [38] *Suppose $A$ is an $n \times n$ matrix. The minimal polynomial of $A$ is the unique monic polynomial $p$ of smallest degree such that $p(A) = 0$. Here, the monic polynomial is a polynomial whose highest-degree coefficient equals* 1.

## 3. The characterization of synchronous states

In this section, our goal is to point out the connection between synchronous trajectories and synchronous states, and then provide the characterization of the synchronous states for heterogeneous linear networks with distinct inner coupling matrices.

For homogeneous linear networks with identical inner coupling matrices [39], it is known that if a network achieves synchronization, then for any synchronous trajectory, there exists a synchronous state such that the synchronous trajectory converges to it. For heterogeneous linear networks with distinct inner coupling matrices, we can also get a similar conclusion as follows.

**Proposition 1.** *Suppose that network (1) synchronizes, then for any synchronous trajectory $(x_1(t), \ldots, x_N(t))$, there must exist a synchronous state $s(t)$ such that $x_i(t) \to s(t)$ when $t \to +\infty$, $i = 1, \ldots, N$.*

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