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Practice article

# Continuous fractional-order Zero Phase Error Tracking Control

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ABSTRACT

A continuous time fractional-order feedforward control algorithm for tracking desired time varying input signals is proposed in this paper. The presented controller cancels the phase shift caused by the zeros and poles of controlled closed-loop fractional-order system, so it is called Fractional-Order Zero Phase Tracking Controller (FZPETC). The controlled systems are divided into two categories i.e. with and without non-cancellable (non-minimum-phase) zeros which stand in unstable region or on stability boundary. Each kinds of systems has a targeted FZPETC design control strategy. The improved tracking performance has been evaluated successfully by applying the proposed controller to three different kinds of fractional-order controlled systems. Besides, a modified quasi-perfect tracking scheme is presented for those systems which may not have available future tracking trajectory information or have problem in high frequency disturbance rejection if the perfect tracking algorithm is applied. A simulation comparison and a hardware-in-the-loop thermal peltier platform are shown to validate the practicality of the proposed quasi-perfect control algorithm.


## 1. Introduction

In the last few decades, the fractional calculus has attracted lots of attention in many research fields, such as physics [1], mechatronics systems [2,3], signal processing [4], biological system [5], chemistry [6], etc. Among them, fractional-order (FO) control system theory and application developed even faster. A lot of significant work aiming at FO controller design algorithm has been done in recent years [7–11]. However, most of the related studies focused on the design of FO feedback controllers. Normally, feedback controllers are aiming at regulation against disturbance input. But when perfect tracking performance is required in control loop, the FO feedforward controller will also be necessary [12].

There are mainly two kinds of trajectory tracking control: one is for tracking a given desired trajectory, and the other is for tracking an unknown trajectory [13]. In this paper, we focus on the former one with two FO controllers (i.e. a feedforward FO controller and a predesigned feedback FO controller) in the system to be controlled. The feedback FO controller is used for dynamic system performance regulation and the FO feedforward controller helps in achieving better tracking performance. The feedforward controller proposed in this paper is designed based on the system inversion theory [14]. For minimum phase systems without non-cancellable zeros and poles (zeros and poles which stand in unstable region or on stability boundary) in their closed-loop transfer functions, the inversed feedforward controller is easy to be designed. However, the design process turns to be complicated when the controlled system has non-cancellable zeros. The non-cancellable zeros will turn into unstable poles after inversion and make the whole system unstable or oscillating. This complicated inversion problem can be solved by an effective tracking control algorithm named Zero Phase Error Tracking Control (ZPETC). The ZPETC tracking algorithm which eliminated the phase error caused by non-cancellable zeros and realized a perfect tracking was put forward by Tomizuka [15]. Ever since, several effective feedforward tracking control algorithms have been proposed based on ZPETC. Torfs et al. gave more insight into ZPETC and compensated the gain error by adding additional feedforward terms [16]; Haack and Tomizuka discussed about inserting a filter before feedforward controller to improve tracking performance in Ref. [17]; a similar work which assumed there were some slowly varying parts in the closed-loop system transfer function was studied by Tsao and Tomizuka in Ref. [18]; an improved ZPETC designed without factorization of zero polynomial was proposed by Adnan et al. [19].

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However, the existing ZPETC related tracking algorithms are mainly used for integer-order (IO) systems and they are unavailable to the rapid developed FO systems. In this paper, we focus on the design of continuous time Fractional-Order Zero Phase Error Tracking Controller (FZPETC) which can be applied on both IO and FO systems. Conventional ZPETC was generated in discrete-time domain, and had to be converted into continuous domain when it was applied on continuous systems. Nevertheless, thanks to the fast computational tools today, a lot of discrete time systems can be replaced by continuous time ones with high sampling frequencies [13]. Especially for FO control systems, more continuous time system transfer functions are used. This is because the conversion of FO transfer functions from continuous time domain to discrete time domain is more complicated and may not be so accurate compared with IO systems.

The FZPETC proposed in this paper is essentially a differentiator or high pass filter whose numerator order is larger than its denominator order. Therefore, future desired trajectory information is needed in the controller design process. The length of the required future desired trajectory decides by the zero locations and design specifications of the controlled system. The controlled system with the proposed feedforward FZPETC and a predesigned feedback controller is depicted in Fig. 1. The method of distinguishing cancellable zeros and non-cancellable ones and the tuning methods of FZPETC aiming at different circumstances are presented respectively. Moreover, for systems with unavailable future tracking trajectory information or having high frequency disturbance rejection problem when the perfect tracking algorithm is applied, an alternative quasi-perfect tracking control algorithm is also presented.

The following of this paper is organized as: in section 2, the stability analysis method of FO control systems is given; the detailed tuning rules of the proposed controller aiming at FO systems with cancellable zeros, with zeros on stability boundaries and with non-cancellable zeros are presented respectively in section 3; a quasi-perfect tracking algorithm is presented in section 5 for the systems which cannot satisfy the perfect tracking requirements or have problem in high frequency gain error; simulation and hardware-in-loop experiment of tracking performance of FO systems are presented in section 4 and 6 to illustrate the effectiveness of the proposed control strategy; finally, the conclusions are made in section 7.

## 2. Stability of fractional order system

Before discussing about the tuning rules of FZPETC, one important step is making clear of how to distinguish cancellable zeros and non-cancellable zeros in the original closed-loop system. Compared with IO control systems, the stability analysis of FO systems is quite different [20]. Several pioneer works have discussed about the stability conditions of FO control systems [20–24]. A stability analysis method of distributed parameter FO system with distributed delay has been given in Ref. [21]; stability analysis of fractional differential system using co-prime factorization algorithm was shown in Ref. [22]; in Ref. [23], the stability conditions for interval FOLTI (Fractional-Order Linear Time Invariant) system have been discussed; a numerical investigation of robust stability of FO uncertain system was discussed in Ref. [25]; the general robust stability conditions for commensurate order FO linear and nonlinear systems were proposed in Ref. [20]. Some of these stability analysis methods were put forward for specific kinds of FO systems, so an appropriate method should be chosen according to the controlled FO system. In this paper, we use the general FO linear system stability condition proposed in Ref. [20] without loss of generality.

An FO control system can be generally described by the following transfer function:

$$G(s)=\frac{b_0 s^{\beta_0}+b_1 s^{\beta_1}+\cdots+b_m s^{\beta_m}}{a_0 s^{\alpha_0}+a_1 s^{\alpha_1}+\cdots+a_n s^{\alpha_n}}=\frac{N(s)}{D(s)}, \tag{1}$$

where, $a_0$, $a_1$,..., $a_n$ and $b_0$, $b_1$,...$b_m$ are constants which represent the coefficients of denominator and numerator; $\alpha_0$, $\alpha_1$,..., $\alpha_n$ ($\alpha_0<\alpha_1<\cdots<\alpha_n$) and $\beta_0$, $\beta_1$,..., $\beta_m$ ($\beta_0<\beta_1<\cdots<\beta_m$) are arbitrary real number orders of denominator and numerator respectively.

The incommensurate order system in Equation (1) can be transformed into a commensurate one as [26]:

$$G'(s)=\frac{b_0+b_1 s^{1/\mu}+\cdots+b_m s^{m/\mu}}{a_0+a_1 s^{1/\mu}+\cdots+a_n s^{n/\mu}},\quad (\mu>1). \tag{2}$$

It should be remarked here that most FO systems can be expressed as Equation (2) [20]. The definition of $G'(s)$ has one Riemann surface with $\mu$ Riemann sheets [27].

Generally, set $\omega=s^{1/\mu}$, then the transfer function with operator $s$ in $s$-domain can be transformed into a complex $\omega$-domain with $\mu$ sheets in Riemann surface [20]. The original principal sheet of Riemann surface is defined as $-\pi<\arg(s)<\pi$. However, after the mapping of $\omega=s^{1/\mu}$, the corresponding principal sheet transforms into $-\pi/\mu<\arg(\omega)<\pi/\mu$. Namely, the right half unstable boundary in $s$-domain becomes $-\pi/2\mu<\arg(\omega)<\pi/2\mu$ in $\omega$-domain. That is to say, in $s$-domain, a stable system will not have right half poles. But in $\omega$-domain, right half poles may exist in stable system as shown in Fig. 2.

Consider an FO pseudo-polynomial as:

$$\begin{aligned} D(s)&=c_1 s^{q_1}+c_2 s^{q_2}+\cdots+c_k s^{q_k}\\ &=c_1 s^{u_1/\mu}+c_2 s^{u_2/\mu}+\cdots+c_k s^{u_k/\mu}\\ &=c_1 (s^{1/\mu})^{u_1}+c_2 (s^{1/\mu})^{u_2}+\cdots+c_k (s^{1/\mu})^{u_k}\end{aligned}, \tag{3}$$

where, $q_i$ $(i=1,2,\ldots,k)=\mu_i/\mu$ $(i=1,2,\ldots,k)$ and $1/\mu$ is the greatest common divisor of $q_i$ [28]. Hence, the fractional degree (FDEG) of polynomial $D(s)$ is got as [28]:

$$\mathrm{FDEG}\{D(s)\}=\max\{\mu_1,\mu_2,\ldots,\mu_k\}.$$

Then, the number of roots of $D(s)$ in Equation (3) can be got from the following proposition:

**Proposition 1.** [29]: Let $D(s)$ be an FO polynomial with $\mathrm{FDEG}\{D(s)\}=n$, then the equation $D(s)=0$ has exactly $n$ roots on the Riemann surface.

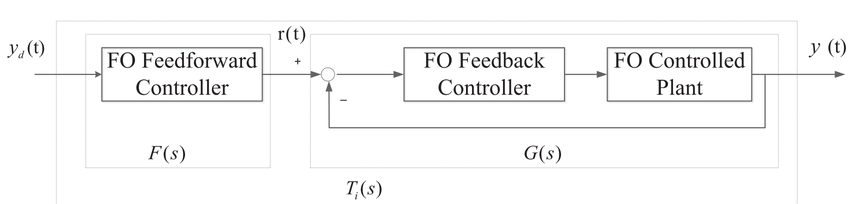


**Fig. 1.** Fractional order control system with feedforward and feedback controllers.

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