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Research article

Decentralized adaptive control of interconnected nonlinear systems with unknown control directions

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ABSTRACT

In this paper, we propose a decentralized adaptive control scheme for a class of interconnected strict-feedback nonlinear systems without *a priori* knowledge of subsystems' control directions. To address this problem, a novel Nussbaum-type function is proposed and a key theorem is drawn which involves quantifying the interconnections of multiple Nussbaum-type functions of the subsystems with different control directions in a single inequality. Global stability of the closed-loop system and asymptotic stabilization of subsystems' output are proved and a simulation example is given to illustrate the effectiveness of the proposed control scheme.

1. Introduction

Controller design without *a priori* knowledge of the control direction is an important problem in control engineering. Since the pioneering work [1], lots of control schemes for linear or nonlinear systems without *a priori* knowledge of the control direction have been proposed, e.g., [2–5]. Control of a single nonlinear system with an unknown control direction has been well addressed by using a Nussbaum function [1]. For example, in Ref. [2], a systematic procedure is developed for designing global adaptive control of a class of strict feedback nonlinear systems with unknown control direction. In Ref. [4], a robust adaptive control approach for a class of time-varying uncertain nonlinear systems in the strict feedback form with completely unknown time-varying virtual control coefficients, uncertain time-varying parameters and unknown time-varying bounded disturbances is proposed.

However, control design for multiple interconnected systems with unknown and different control directions is much trickier. In this paper, we investigate the adaptive control of multiple interconnected high-order nonlinear systems with unknown control directions. The decentralized control [6–14] is effective for interconnected systems and has been an active research in the control community. But the decentralized adaptive control for interconnected nonlinear systems with unknown control directions is quite difficult since the interconnections may trigger instability and the unknown control directions make the analysis complicated.

A few results have been obtained for the decentralized adaptive control of interconnected nonlinear systems with unknown control directions. In Ref. [14] a decentralized adaptive control scheme for a class of linear systems with unknown control directions is proposed. In Ref. [15] a decentralized adaptive output-feedback stabilizer for a class of large-scale nonlinear time-delay systems with unknown control directions is proposed. However, in Refs. [14] and [15], the subsystems as well as the interconnections among the subsystems are sophisticatedly assumed to be under certain conditions such that the stability analysis could be done separately for each subsystem, without considering the interconnections of multiple Nussbaum-type functions. Therefore normal Nussbaum-type functions could be used here. This means the control schemes in Refs. [14] and [15] could not be applied to more general interconnected nonlinear systems. In Ref. [20] a decentralized approximation-free control design approach is proposed for interconnected nonlinear time-delay systems with unknown nonaffine pure-feedback nonlinearities but it has to be assumed that the interconnections among the multiple systems are strictly bounded. In Ref. [21] the decentralized output feedback control problem is considered for a class of large-scale systems with unknown time-varying delays, but similar to [15], strict assumptions have to be made on the nonlinearity of the subsystems

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as well as the interconnections. In Ref. [22] a saturated Nussbaum function based approach for robotic systems with unknown actuator dynamics is presented, but the proposed Nussbaum function could only be used in a single system.

The major difficulty of decentralized adaptive control for interconnected nonlinear systems with unknown control directions is that, if the assumptions in Refs. [14] and [15] are not made, stability analysis of the closed-loop interconnected systems requires quantifying the interconnections of multiple Nussbaum functions in a single inequality, and when using common Nussbaum function, e.g., Nussbaum functions used in Refs. [2–4], the quantification still remains unknown. To the best of our knowledge, no decentralized adaptive control scheme has so far been proposed for interconnected high order strict feedback nonlinear systems with unknown subsystems control directions. In this paper, we propose a decentralized adaptive control scheme for interconnected high-order nonlinear systems without *a priori* knowledge of control directions by proposing a novel Nussbaum-type function. The novel Nussbaum function is specially designed such that quantifying the addition of multiple Nussbaum functions is possible, thus the assumptions in Ref. [15] are not needed. The major contribution of this paper can be summarized as follows: 1) A novel Nussbaum function is proposed in this paper such that the interconnections of multiple Nussbaum-type functions with different control directions in a single inequality could be quantified, which gives great convenience to the controller design and stability analysis. 2) Asymptotic stabilization control is achieved in the presence of external disturbances and uncertain parameters.

The paper is organized as follows. In Section II, we formulate the control problem. In Section III, we propose a novel Nussbaum-type function and a key theorem is drawn. In section IV, we propose a decentralized adaptive controller and analyze the closed-loop stability. In Section V, a simulation example is given and finally the paper is concluded in Section VI.

2. Problem formulation

We consider an interconnected nonlinear system consisting of N subsystems modelled as:

$$\begin{aligned}\dot{x}_{i,k} &= x_{i,k+1} + \theta_i(t) f_{i,k}(\overline{x}_{i,k}) + \psi_{i,k}(y_1, \ldots, y_N, t)\\ \dot{x}_{i,n_i} &= b_i u_i + \theta_i(t) f_{i,n_i}(\overline{x}_{i,n_i}) + \psi_{i,n_i}(y_1, \ldots, y_N, t) + d_i(t)\\ y_i &= x_{i,1}\end{aligned} \tag{1}$$

where $i = 1, \ldots, N$, b_i denotes the unknown control coefficient of subsystem i with an unknown sign, $\overline{x}_{i,k} = [x_{i,1}, \ldots, x_{i,k}]^T$, $k = 1, \ldots, n_i - 1$, u_i and y_i are the states, input and output of ith subsystem respectively. $\theta_i(t) \in \Re^{\nu_i}$ are unknown bounded time-varying parameters, $f_{i,k}$ are known smooth functions, $\psi_{i,k}$ are unknown interactions among subsystems, $d_i(t)$ is the external disturbance assuming that $|d_i(t)| \le D_i$ where D_i is an unknown constant. The control objective is to make the subsystem output y_i stabilized. For b_i and the interconnections of the subsystems, we made the following assumptions.

Assumption 1. b_i is an unknown constant and the sign of b_i, i.e., $\mathrm{sign}(b_i)$, is also unknown. Meanwhile, it has $b_i \neq 0$. Denote $b_{\max} = \max\{|b_1|, \ldots, |b_N|\}$ and $b_{\min} = \min\{|b_1|, \ldots, |b_N|\}$, then there exists an arbitrary known constant χ such that $\chi \ge \frac{b_{\max}}{b_{\min}}$.

Assumption 2. The unknown functions $\psi_{i,k}$, $k = 1, \ldots, n_i$ satisfy

$$\psi_{i,k}^2(y_1, \ldots, y_N, t) \le \sum_{j=1}^{N} \varrho_{i,k,j} |y_j| \phi_{i,k,j}(y_j) \tag{2}$$

where $\varrho_{i,k,j} \ge 0$ are unknown constants and $\phi_{i,k,j}(y_j) \ge 0$ are known smooth functions.

Remark 1. Assumption 1 implies that the control coefficients $b_i(t)$ are either strictly positive or strictly negative, which is a necessary condition for the system (1) to be controllable. Assumption 2 means the unknown interconnection function $\psi_{i,k}$ is bounded by some known smooth function $\phi_{i,k,j}(y_j)$. This is a mild and reasonable assumption since in reality partial knowledge of $\psi_{i,k}$ could be known by modelling and measurement.

Remark 2. It can be observed from (1) that all system parameters are allowed to be time-varying and interactions exist in every equation of each subsystem, which significantly enlarges the application of the proposed control scheme. Assumption 2 is similar to assumption 4 in Ref. [19].

3. A novel Nussbaum function

A Nussbaum-type function $\mathcal{N}(\cdot)$ is the one with the following properties [10]:

$$\begin{aligned}&\lim_{k\to\infty} \sup \frac{1}{k} \int_0^k \mathcal{N}(\tau) d\tau = \infty,\\ &\lim_{k\to\infty} \inf \frac{1}{k} \int_0^k \mathcal{N}(\tau) d\tau = -\infty.\end{aligned} \tag{3}$$

Commonly used Nussbaum-type functions include $k^2\cos(k)$, $e^{k^2}\cos(k)$, $k^2\sin(k)$. For the existing Nussbaum-type functions, it is still unclear how to analyze the interactions of the coexisting Nussbaum-type functions in the same inequality, as shown later in (9). To overcome this difficulty, we propose the following novel Nussbaum-type function:

$$\mathcal{N}_i(k_i) = \frac{\alpha^2\beta^{2i} + 1}{\beta^i\sqrt{\alpha^2\beta^{2i} + 1}} e^{\alpha|k_i|} \sin\left(\frac{k_i}{\beta^i}\right) \tag{4}$$

where $i = 1, \ldots, N$, α and β are positive constants.

Lemma 1. *$\mathcal{N}_i(k_i)$ is an odd function. Let $G_i(k_i) = \int_0^{k_i} \mathcal{N}_i(\tau)d\tau$, then it is obvious that $G_i(k_i)$ is an even function. By direct calculation, we have*

$$\int_0^{k_i} \mathcal{N}_i(\tau)d\tau = e^{\alpha k_i} \sin\left(\frac{k_i}{\beta^i} - \epsilon_i\right) \tag{5}$$

for $k_i > 0$, where $\epsilon_i = \arccos\left(\frac{\alpha\beta^i}{\sqrt{\alpha^2\beta^{2i}+1}}\right)$. □

In the rest of the paper, we only consider the case when $k_i > 0$, and for $k_i < 0$, it could be analyzed similarly. For clarity, let $G_i^{b_i}(k_i) = \int_0^{k_i} b_i \mathcal{N}_i(\tau)d\tau = b_i e^{\alpha k_i} \sin(\frac{k_i}{\beta^i} - \epsilon_i)$, where $\mathrm{sign}(b_i) = 1$ or $\mathrm{sign}(b_i) = -1$, clearly $G_i^{-b_i}(k_i) = -G_i^{b_i}(k_i)$.

Lemma 2. *Let $\beta = \frac{1}{M}$, where $M > 4$ is a positive integer, then there exist periodical intervals $[\underline{a}_j, \bar{a}_j]$, $j = 1, \ldots, \infty$ such that $\forall x \in [\underline{a}_j, \bar{a}_j]$, $\mathrm{sign}(b_i)\sin(\frac{x}{\beta^i} - \epsilon_i) < 0$ and thus $G_i^{b_i}(x) < 0$, $i = 1, \ldots, N$.*

Proof. Without loss of generality, assuming $\mathrm{sign}(b_i) = 1$, then $G_i(x)$ is negative in $x \in [\frac{2n\pi + \pi + \epsilon_i}{M^i}, \frac{2n\pi + 2\pi + \epsilon_i}{M^i}]$. Meanwhile, $G_{i+1}(x)$ is negative in $x \in [\frac{2m\pi + \pi + \epsilon_i}{M^{i+1}}, \frac{2m\pi + 2\pi + \epsilon_i}{M^{i+1}}]$ if $\mathrm{sign}(b_{i+1}) = 1$ or in $x \in [\frac{2m\pi + \epsilon_i}{M^{i+1}}, \frac{2m\pi + \pi + \epsilon_i}{M^{i+1}}]$ if $\mathrm{sign}(b_{i+1}) = -1$, as shown schematically in Fig. 1. Let $m = n * M + g$ where g is a positive integer satisfies

$$\begin{aligned}(2g+1)\pi + \epsilon_{i+1} &> M\pi + M\epsilon_i\\ (2g+2)\pi + \epsilon_{i+1} &< 2M\pi + M\epsilon_i\end{aligned} \tag{6}$$

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