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Research article

A simple tuning method of fractional order PI^{λ}-PD^{μ} controllers for time delay systems

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A R T I C L E I N F O

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A B S T R A C T

In this paper, a practical tuning technique is presented to obtain all stabilizing fractional order PI^{λ}-PD^{μ} controller parameters ensuring stability for processes with time delay using the stability boundary locus and the weighted geometrical center (WGC) methods. The method is based on obtaining of stability regions plotted by using the stability boundary locus in the (k_d, k_f)-plane and (k_p, k_i)-plane, and then computing the weighted geometrical centers of these regions. After obtaining PD^{μ} controller parameters using the WGC method from the stability region, desired PI^{λ} controller parameters are computed by the same procedure. This paper provides a simple and efficient tuning method to obtain stabilizing parameters of PI^{λ}-PD^{μ} controller for time delay systems. The important advantages of the method are both calculating of controller parameters without using any complex solution methods and ensuring the stability of closed loop system. Illustrative examples are given to demonstrate the benefits and the simplicity of the proposed method.

1. Introduction

PID (Proportional-Integral-Derivative) controllers are one of the most important controller algorithm in industrial processes. Due to their simple structure, they are still widely used in practice although control theory has been developed significantly [1,2]. Since the derivative action is not used very often, control loops are mostly PI (Proportional-Integral) [3]. Many studies have been carried out to determine PID controller parameters [1,2,4–6]. Although PID controllers have an acceptable control performance for many open loop stable processes, in general, good closed-loop performance cannot be achieved for control of integrating, unstable and resonant processes because of structural limitations of PID. And, unit step responses are usually achieved with a high overshoot and oscillation [7,8]. On the other hand, it is approved that a PI-PD (Proportional Integral-Proportional Derivative) controller which has four parameters to be adjusted can be taught as modified form of PID controller and provides very satisfactory closed-loop performances for integrating, unstable and resonant processes [7–9]. Many important tuning studies for PI-PD and PID controllers have been recently reported in Refs. [10–16]. The need to control processes with time delay that can cause oscillations or instability can be found in many aspects of industrial systems. It is difficult to control of processes with time delay using conventional control methods. Therefore, modeling and stability analysis of such systems is very important.

Fractional order systems and their applications are one of the most leading studies today. Much more attention has been given to fractional calculus and its applications by many researchers for the last four decades [17–19]. Fractional calculus has been developed mainly as a pure mathematical discipline for three centuries [17,18]. It has been used for the description of dynamic behavior of various physical systems and real materials in the last two decades. Using of fractional derivatives and integrals is an excellent instrument for the description of memory and hereditary properties [17–19]. One of the most important properties of fractional order representation is more accurate to describe real world systems than those of integer order models since the integer order representation may lead to significant differences between the mathematical model and real system [20,21]. A control system may have both fractional order dynamics and be controlled by fractional order controller. For example, $PI^{\lambda}D^{\mu}$ controller, PI^{λ} controller, PD^{μ} controller etc. [22]. Some various design methods for $PI^{\lambda}D^{\mu}$ controllers have been developed, recently [21,23–35]. Besides, some important results related to fractional order systems and their applications have been reported in Refs. [36–40].

As PID controllers are still widely used in industrial systems, the extension of conventional integer order PID controller based on

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fractional calculus can be seen as important controllers for future studies. Thus, fractional order PI$^{\lambda}$-PD$^{\mu}$ controller can also be considered as an important controller structure. In this paper, PI$^{\lambda}$-PD$^{\mu}$ controller is introduced. It can be considered as a generalization of the regular PI-PD controller. It has been shown that the PI$^{\lambda}$D$^{\mu}$ controllers provide better performance than those of integer order controllers for both of integer order and fractional order systems due to extra tuning parameters λ and μ [21,22]. Since PI-PD controller provides better performance than PID, it can be said that PI$^{\lambda}$-PD$^{\mu}$ controller will provide better performance results than conventional PI-PD controller. However, obtaining PI$^{\lambda}$-PD$^{\mu}$ controller parameters will be extremely difficult due to the fact that we have to deal with six parameters, k_f, k_d, k_p, k_i, λ and μ, for tuning. As stated before, many important results about the fractional order PID (FO-PID) controllers have been recently reported. However, many of them have only focused on the tuning of PI$^{\lambda}$D$^{\mu}$, PI$^{\lambda}$ or PD$^{\mu}$ controllers. To the best knowledge of the author of this paper, studies related to PI$^{\lambda}$-PD$^{\mu}$ controllers are not extensive. A few of the existing studies have mainly focused on the one part of fractional order PI-PD controller, ie., they are concerned with PI$^{\lambda}$—PD [33,41]. In this study, both parts of the PI$^{\lambda}$-PD$^{\mu}$ controller are considered as fractional order. The studies related to fractional order controllers mainly require complex solution methods or could not provide a practical solution in terms of selecting controller parameters. Thus, the extension of the studies providing simple and practical tuning methods for fractional order PI$^{\lambda}$-PD$^{\mu}$ controller design will be very important. In this study, not only fractional order PI$^{\lambda}$-PD$^{\mu}$ controller is introduced, but also a method providing very good results for PI$^{\lambda}$-PD$^{\mu}$ controller design is presented. This paper presents an effective and simple numerical tuning algorithm of fractional order PI$^{\lambda}$-PD$^{\mu}$ controllers for time delay systems.

An example of PI$^{\lambda}$-PD$^{\mu}$ controller structure is shown in Fig. 1. Thanks to the internal PD$^{\mu}$ feedback loop, an open loop unstable system can be converted into an open loop stable system and appropriate position of stable open loop poles is ensured [9]. As stated before, PI$^{\lambda}$-PD$^{\mu}$ controller has advantages over the conventional PI-PD controller since it has more flexibility due to extra tuning parameters λ and μ. However, tuning of PI$^{\lambda}$-PD$^{\mu}$ controller is extremely difficult task since it has six parameters to determine. Therefore, it is very important to obtain stabilizing parameters of PI$^{\lambda}$-PD$^{\mu}$ controller. This paper provides a simple and practical tuning method to obtain PI$^{\lambda}$-PD$^{\mu}$ controller parameters ensuring the stability.

The proposed method is based on a fairly simple technique. The stability regions are obtained after the selection of λ and μ, which are the parameters of fractional-order controller. Then the weighted geometrical center points for these stability regions are determined. Although the choice of λ and μ is experimentally done, these points mostly provide very good performance results. But these values must be within reasonable limits. They can be considered between 0.5 and 1.5 to obtain good results. Since the proposed method does not require any extra tuning features, it is very simple and practical. The examples show that this simple tuning method performs quite reliable results.

This paper is organized as follows: Stability regions of PI$^{\lambda}$-PD$^{\mu}$ controller by using the stability boundary locus method are obtained in Section 2. Then, the weighted geometrical center method is introduced. To illustrate the efficiency of the proposed method, some simulation examples are also given. Finally, concluding remarks and discussion for the future studies are given in the last section.

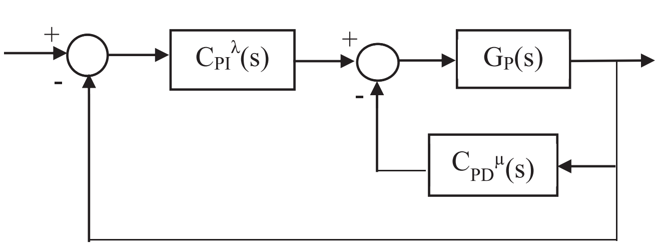

Fig. 1. A control system with fractional order PI$^{\lambda}$-PD$^{\mu}$ controller.

2. Obtaining stability regions of fractional order PI$^{\lambda}$-PD$^{\mu}$ controller

As shown in Fig. 1, $G_P(s)$ is the plant to be controlled and it is given by Eq. (1).

$$G_P(s) = \frac{N_P(s)}{D_P(s)} e^{-\tau s} \tag{1}$$

$C_{PI^{\lambda}}(s)$ and $C_{PD^{\mu}}(s)$ are the fractional order PI (FO-PI) and fractional order PD (FO-PD) controllers respectively and they are defined by Eqs. (2) and (3) (0<λ,μ<2).

$$C_{PD^{\mu}}(s) = k_f + k_d s^{\mu} \tag{2}$$

$$C_{PI^{\lambda}}(s) = k_p + \frac{k_i}{s^{\lambda}} = \frac{k_p s^{\lambda} + k_i}{s^{\lambda}} \tag{3}$$

Here, the main problem is to obtain all PI$^{\lambda}$-PD$^{\mu}$ controller parameters that make closed loop system stable. This study has been carried out with the aim of obtaining PI$^{\lambda}$-PD$^{\mu}$ parameters ensuring the stability of system.

2.1. Stability region of FO-PD controller

Consider the internal feedback loop with FO-PD (PD$^{\mu}$) controller of Fig. 1, the characteristic equation is obtained by Eq. (4)

$$\Delta_{PD^{\mu}}(s) = 1 + C_{PD^{\mu}}(s) G_P(s) \tag{4}$$

Using Eqs. (1) and (2), this expression can be written as follows

$$\Delta_{PD^{\mu}}(s) = D_P(s) + \left(k_f + k_d s^{\mu}\right) N_P(s) e^{-\tau s} \tag{5}$$

By using $s = j\omega$ into Eq. (5), following equation is obtained.

$$\Delta_{PD^{\mu}}(j\omega) = D_P(j\omega) + \left(k_f + k_d (j\omega)^{\mu}\right) N_P(j\omega) e^{-\tau j\omega} \tag{6}$$

In the parameter space approach, a stable polynomial root can cross over the imaginary axes in three possibilities: at zero (the real root boundary), at infinity (the infinite root boundary), and at a finite number of singular frequencies (the complex root boundary). The details can be seen in Refs. [27,42–45].

(i) *Real Root Boundary (RRB):* When a real root of characteristic polynomial crosses over the imaginary axis at $s = 0$, the real root boundary is obtained by substituting $j\omega = 0$ in $\Delta(j\omega)$ of Eq. (6).
(ii) *Infinite Root Boundary (IRB):* When a real root crosses over the imaginary axis at infinity, the IRB can be characterized by taking $s = \infty$ in Eq. (6).
(iii) *Complex Root Boundary (CRB):* When a pair of complex roots of characteristic polynomial cross over the imaginary axis at $s = j\omega$, the system becomes unstable. Thus, real and imaginary parts of $\Delta(s)$ in Eq. (6) become zero simultaneously. This complex root boundary is also called the stability boundary locus. And, it is obtained by the procedure presented as follows.

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