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Practice article

Robust feedback linearization for nonlinear processes control

José de Jesús Rubio

Sección de Estudios de Posgrado e Investigación, Esime Azcapotzalco, Instituto Politécnico Nacional, Av. de las Granjas no. 682, Col. Santa Catarina, México D.F., 02250, Mexico

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ABSTRACT

In this research, a robust feedback linearization technique is studied for nonlinear processes control. The main contributions are described as follows: 1) Theory says that if a linearized controlled process is stable, then nonlinear process states are asymptotically stable, it is not satisfied in applications because some states converge to small values; therefore, a theorem based on Lyapunov theory is proposed to prove that if a linearized controlled process is stable, then nonlinear process states are uniformly stable. 2) Theory says that all the main and crossed states feedbacks should be considered for the nonlinear processes regulation, it makes more difficult to find the controller gains; consequently, only the main states feedbacks are utilized to obtain a satisfactory result in applications. This introduced strategy is applied in a fuel cell and a manipulator.

1. Introduction

Robust feedback linearization technique is concerned with the local stability of a nonlinear process. It is a formalization of the intuition that a nonlinear process should behave similarly to its linearized approximation for small range motions. Because all physical processes are inherently nonlinear, robust feedback linearization technique serves as the fundamental justification of using linear control strategies in practice, i.e., it shows that stable design by linear control assures stability of the original physical process locally. This research is focused in this interesting issue.

There are some investigations about stable controllers. Stability of controllers for delayed processes is introduced in Refs. [1–3]. In Refs. [4–7], stability of some kind of adaptive controls is mentioned. Stability of controllers for linear processes is considered in Refs. [8–11]. The above mentioned investigations show that stable controllers could be directly designed for nonlinear processes; however, in some cases, stable controllers are employed in synthetic models, which is a little far to applications.

There is some research about robust control of linearized models. Controllers based on a feedback linearization are designed in Refs. [12–15]. In Refs. [16–19], processes controls based on linearized models are designed. Robust feedback linearization is mentioned in Refs. [20–23]. In Refs. [24–27], controllers of the linearized turbine, pendulum, robotic arm, and rotor are investigated. The aforementioned research shows that in these days a robust linearization technique is utilized in applications; therefore, it is an actual and interesting issue.

Robust feedback linearization technique is used for the nonlinear processes regulation. Regulation is a kind of control in which all process states should converge to constant references. Robust feedback linearization technique has two main problems which are focused on differences between the theory and applications:

1. Theory says that if a linearized controlled process is stable, then nonlinear process states are asymptotically stable, it means that all process states should converge to zero [28,29]. Nevertheless, it is not exactly satisfied in applications because in some cases, some nonlinear process states only converge to small values.
2. A main state is when a state is utilized by the controller for regulation of the same state, while a crossed state is when a state is utilized by the controller for regulation of a different state. Theory says that feedback of all the main and crossed states should be considered in the controller for the nonlinear process regulation [28,29]. It sometimes makes more difficult to find the controller gains.

This investigation proposes a strategy to solve aforementioned problems which is detailed by the following two steps:

1. From Lyapunov theory, uniform stability is stronger than stability because the first is satisfied for any initial time, while the second is satisfied only for a zero initial time. However, uniform stabil-

E-mail address: jrubioa@ipn.mx.

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ity is weaker than asymptotic stability because the first assures the error convergence to a small value, while the second assures the error convergence to zero. This study suggests a theorem to prove that if the linearized controlled process is stable, then nonlinear process states are uniformly stable. It represents better the control applications.

2. In the introduced method, only main states feedbacks are utilized by the controller to obtain a satisfactory result in applications. It makes easier to find the controller gains.

From the above research, nonlinear processes with robustness are studied in Refs. [13,14,16,18,23,30,31]. In control theory, robust control is an approach that explicitly deals with uncertainty. Robust methods aim to achieve robust performance or stability in presence of uncertainties. In results, this method is applied to two nonlinear processes with inputs and parameters uncertainties.

Finally, the suggested technique is applied to two nonlinear processes: a fuel cell and a manipulator. A fuel cell is applied for the electricity generation from hydrogen fuel. A manipulator is mainly utilized to move objects in the automobile industry.

Other sections are focused in the following issues. In Section 2, the proposed controller is introduced for the nonlinear processes regulation. Suggested controller is applied for the fuel cell regulation in Section 3. In Section 4, mentioned controller is applied for the manipulator regulation. Control results for the two processes regulation are shown in Section 5. In Section 6, the conclusion and future research are described.

2. Controller of nonlinear processes

In this section, nonlinear models are presented, a robust feedback linearization controller is proposed, and a theorem to study the process stability is introduced.

2.1. Nonlinear processes

In this subsection, nonlinear processes are described.

Consider the following nonlinear processes:

$$\dot{X} = f(X, U) \tag{1}$$

where $X \in \mathfrak{R}^n$ are states, $U \in \mathfrak{R}^m$ are inputs, and $f(\cdot) \in \mathfrak{R}^n$ are continuous differentiable nonlinear functions.

2.2. Proposed controller

In this subsection, a controller for the nonlinear processes regulation is studied. The objective of controller is that using inputs, states of nonlinear processes should follow constant references, it is denoted as the states regulation.

Consider control functions as follows:

$$U = -KX \tag{2}$$

where $K \in \mathfrak{R}^{m \times n}$ are controller gains.

Fig. 1 shows a proposed controller where U are inputs, X are states, and $f(\cdot)$ are nonlinear functions.

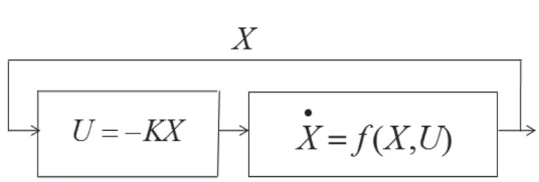

Fig. 1. A proposed controller.

2.3. Stability analysis

In this subsection, stability of an introduced controller for the nonlinear processes regulation is analyzed. It is based on four parts, 1) a closed loop nonlinear process is obtained, 2) the controllability is studied, 3) controller gains are determined, and 4) a theorem to analyze the stability of the controller applied to nonlinear processes is introduced.

2.3.1. Closed loop nonlinear process

A closed loop nonlinear process of controller applied for the nonlinear processes regulation will be obtained. It will be used for stability analysis.

Applying Taylor series to (1) gives the following result:

$$\dot{X} = \frac{\partial f(X,U)}{\partial X}\left(X - X_d\right) + \frac{\partial f(X,U)}{\partial U}\left(U - U_d\right) + r \tag{3}$$

where X_d are desired states and U_d are desired inputs, X_d and U_d are considered as zero because it is the regulation case, r is a residue. Adding and subtracting $\left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right)X$ and $\left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right)U$ to (3) gives:

$$\begin{aligned}
\dot{X} &= \frac{\partial f(X,U)}{\partial X}X + \frac{\partial f(X,U)}{\partial U}U + r \\
&\quad + \left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right)X - \left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right)X \\
&\quad + \left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right)U - \left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right)U \\
\Rightarrow \dot{X} &= \left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right)X + \left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right)U \\
&\quad + \left[\frac{\partial f(X,U)}{\partial X} - \left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right)\right]X \\
&\quad + \left[\frac{\partial f(X,U)}{\partial U} - \left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right)\right]U + r
\end{aligned} \tag{4}$$

Equation (4) can be rewritten as follows:

$$\begin{aligned}
\dot{X} &= AX + BU + \widetilde{A}X + \widetilde{B}U + r \\
\Rightarrow \dot{X} &= AX + BU + \delta
\end{aligned} \tag{5}$$

where:

$$\begin{aligned}
A &= \left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right) \\
B &= \left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right) \\
\widetilde{A} &= \frac{\partial f(X,U)}{\partial X} - \left(\frac{\partial f(X,U)}{\partial X}\Big|_{X=0,U=0}\right) \\
\widetilde{B} &= \frac{\partial f(X,U)}{\partial U} - \left(\frac{\partial f(X,U)}{\partial U}\Big|_{X=0,U=0}\right)
\end{aligned} \tag{6}$$

and $\delta = \widetilde{A}X + \widetilde{B}U + r$ is an unmodelled error which is bounded as follows $\|\delta\| \leq \overline{\delta}$.

Substituting the control function (2) in equation (5) gives:

$$\begin{aligned}
\dot{X} &= AX + B[-KX] + \delta \\
\Rightarrow \dot{X} &= A_C X + \delta
\end{aligned} \tag{7}$$

where $A_C = A - BK$. Equation (7) is the closed loop nonlinear process.