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An Evolutionary Strategy For Adaptive Network Control and Synchronization and its Applications

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Abstract: In a recent paper, we proposed an evolutionary strategy for control and synchronization based on an extension of the Edge-Snapping method which was presented earlier in the literature. Here we show the generality of our methodology by applying it to a network of Rossler chaotic oscillators. Finally, we test the robustness properties of the evolutionary strategy, to variations of dynamical and control parameters.

Keywords: Complex systems, adaptation, chaos, synchronization, evolutionary networks, oscillators

1. INTRODUCTION

Evolution is a central feature of many natural systems (Darwin, 1859). It consists of two key ingredients: mutation and selection. The first is based on the recombination within a class of organisms that yields the formation of new species. Then selection is the mechanism determining the survival of the fittest to perform a certain function. Evolutionary algorithms are known in the literature as genetic algoritms (Davis, 1991), useful to optimize the behaviour of a given system with respect to a payoff function, through iterative mutation and selection. Recently, this mechanism has been succesfully used in the field of complex networks (Tan et al., 2014) to explore and uncover mechanisms for the emrgence of collective behaviors in a multi-agent network. Indeed, one of the most challenging open problems in Network Science (Boccaletti et al., 2006) is aimed to uncover the intrinsic relation between structure and function of a complex dynamical network. There are several approaches in literature to address this fundamental problem, taking as a representative example the problem of evolving the network structure to achieve synchronization of coupled oscillators.

First, in Gorochowski et al. (2010) an analysis is provided of how network topology can be evolved to improve the synchronization property through the rewiring of edges of a given network. Numerical simulation are performed using the computational tool called NetEvo which is based on a simulated annealing metaheuristic. The main contribution is to use simulated output from the system to direct the evolution of the network, instead of simply considering topological attributes when assessing current performance. A similar approach, based on Monte Carlo optimization, is proposed in Yanagita and Mikhailov (2010), where the analysis is carried out on a specific class of phase oscillators, i.e. Kuramoto oscillators. Authors show, starting with an initial random network of oscillators with a heterogeneous frequency distribution, that its autonomous synchronization ability can be largely improved by appropriately rewiring the links between the elements.

Other approaches are based on gradient-based methods to enhance the synchronization performance of a network with respect to a given objective function. For instance, in Tanaka and Aoyagi (2008) this optimization is performed in weighted networks of phase oscillators systems, showing that stronger weights tend to be assigned to a connection between two oscillators with greatly different natural frequencies. Moreover in Skardal et al. (2014) an analytical model is derived to study the synchronization performance of a network of linearized Kuramoto oscillators, when varyng the network structure and/or the overall coupling stregth.

However, the Monte Carlo approach is generic and powerful, but it is typically time-consuming and increasingly cumbersome to apply to large-scale networks. Moreover gradient-based methods are effective in giving some analytical model to study the problem, but they assume some constraints to derive the evolution rule of the coupling strength, e.g. usually only linearized dynamics are studied. Also these rules are not local, in the sense that some global information on the entire network is used.

We showed in Scafuti et al. (2015) that, rather than aiming at achieving optimality for network synchronization, it is possible to obtain minimal networks which guarantee frequency synchronization and give a relatively high value of some synchronization measure, by a small number of links. The evolutionary strategy for control and synchronization, proposed in Scafuti et al. (2015), is based on an extension of the Edge Snapping mechanism, firstly introduced in DeLellis et al. (2010). In this paper, we recall that methodology and show its generality. Indeed, even if the method

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is developed in a network of Kuramoto oscillators, here we prove that it also applies to general nonlinear oscillators (e.g. limit cycle or chaotic oscillators). We then show the robustness of our evolutionary method, to variations of dynamical and control parameters.

2. BACKGROUND

2.1 Kuramoto Model

The Kuramoto model, firstly introduced in Kuramoto (1984), has been widely studied in the literature due to the occurence of synchronization phenomena in many natural as well as artificial systems. Examples include biological systems, such as the oscillatory behaviour of neurons and the synchronization of organisms to an external rythm (Strogatz, 2000). Other examples, in the field of engineering, include array of lasers and smart grids (Dorfler et al., 2013). Indeed, all of these examples, can be modeled as an ensemble of oscillators (each with its own natural frequency) that, by means of mutual interactions, tend to oscillate with a common frequency, despite some differencies in the natural frequencies of the individual uncoupled oscilllators. The network model is

$$\dot{\theta}_n = \omega_n + \frac{K}{N}\sum_{m=1}^{N}\sin(\theta_n - \theta_m) \tag{1}$$

where θ_n is the phase of the $n-$th oscillator and ω_n its natural frequency. The second term on the right hand-side is the coupling among neighboring oscillators, while the scalar quantity K represents the strenght of the coupling term. To measure the degree of synchronization, Kuramoto used the so-called order parameter

$$R(t)e^{i\Psi(t)} = \frac{1}{N}\sum_{n=1}^{N} e^{i\theta_n(t)} \tag{2}$$

where i is the imaginary unit. The complex number in (2) represent the centroid of the population. In particular it is easy to see that $R(t) \in [0, 1]$, with $R(t) = 1$ when there is perfect phase synchronization (at the time instant t), while $R(t)$ vanishes when the network is completely desynchronized (i.e. the oscillators are equally spaced on the unitary circle). The order parameter is a time varying quantity and, in general, it may oscillate in time. Nevertheless, there are conditions on K ensuring $R(t)$ locks to a steady-state value. This phenomenon is called phase-locking and represents the situation in which the distance between each couple of oscillators remains constant in time. Moreover, if a phase locked solution exists, then the frequency of each oscillator is equal to the average $\Omega = 1/N\sum_n \omega_n$ of all the natural frequencies.

2.2 Edge snapping

Edge snapping (DeLellis et al., 2010) is an adaptive strategy for the evolution of an unweighted network. Specifically, a second order dynamics is selected for each edge in the network as

$$\ddot{k}_{nm} + d\ \dot{k}_{nm} + \frac{\partial V(k_{nm})}{\partial k_{nm}} = \mathcal{G}(e_{nm}) \tag{3}$$

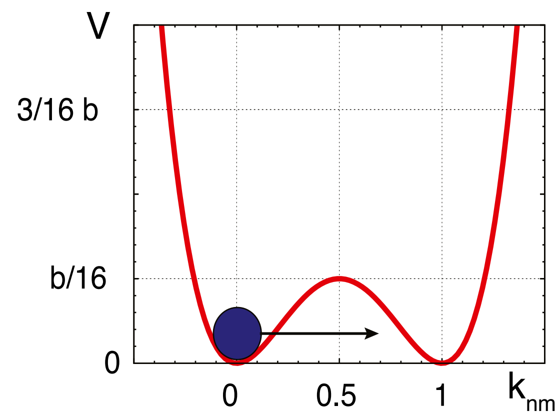

Fig. 1. Double-well potential $V(k_{nm}) = b\ k_{nm}^2(k_{nm}-1)^2$.

where d is a damping factor, e_{nm} is the error between the states of the agent n and m, and $V(k_{nm})$ is a double-well potential. It is chosen as

$$V(k_{nm}) = b\ k_{nm}^2(k_{nm}-1)^2 \tag{4}$$

This smooth bistable potential has two local minima corresponding to the desired equilibria of the system: $k_{nm} = 0$ (non-active edge) and $k_{nm} = 1$ (active edge), while the parameter b is proportional to the height of the barrier between the two equilibria.

At the onset of the evolution, all nodes are disconnected, that is, all edges in the network are at the equilibrium state corresponding to edges being switched off, i.e. $k_{nm} = 0$. Edge snapping is induced as the external forcing (error between neighboring nodes) is strong enough to drive the corresponding gain to the other equilibrium associated to the edge being switched on. As can be noted from (3), network evolution can be controlled by choosing the potential V and the input function $\mathcal{G}$. Moreover, almost all solutions of (3) converge towards $k_{nm} = 0/1$. Specifically, if we exclude the trival case in which $k_{nm}(0) = 1/2\ \forall(n,m)$ and $e_{nm}(0) = 0\ \forall(n,m)$, the network evolves to a final topology, whose structure depends on the initial conditions, the damping d and the parameter b.

The strategy is based on a distributed adaptive nonlinear approach as explained in DeLellis et al. (2010) and is therefore a generic decentralized approach relying only on a nonlinear potential to drive edge adaptation. We wish to emphasize that edge-snapping is an adaptive strategy for the evolution of an unweighted network, subject to constraints. Indeed the barrier of the potential between the two wells, acts as a constraint for edges evolution. As explained above, if the driving force is not strong enough, the edge, after a transient, will remain in the well corresponding to the absence of link. Also, the height of the barrier can be tuned varying the parameter b in the expression of the potential V. The higher the barrier b, the stronger the constraint.

3. EVOLUTIONARY EDGE SNAPPING

In this section we briefly recall the methodology presented in Scafuti et al. (2015). We start by considering a network