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Research article

Robust preview control for a class of uncertain discrete-time systems with time-varying delay

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ABSTRACT

This paper proposes a concept of robust preview tracking control for uncertain discrete-time systems with time-varying delay. Firstly, a model transformation is employed for an uncertain discrete system with time-varying delay. Then, the auxiliary variables related to the system state and input are introduced to derive an augmented error system that includes future information on the reference signal. This leads to the tracking problem being transformed into a regulator problem. Finally, for the augmented error system, a sufficient condition of asymptotic stability is derived and the preview controller design method is proposed based on the scaled small gain theorem and linear matrix inequality (LMI) technique. The method proposed in this paper not only solves the difficulty problem of applying the difference operator to the time-varying matrices but also simplifies the structure of the augmented error system. The numerical simulation example also illustrates the effectiveness of the results presented in the paper.

1. Introduction

Control systems designs require that the outputs track the reference signals without steady-state error, for example, induction motor control [1–3], robot control [4], power system control [5,6]. Preview control can fully utilize the future information of references or disturbances to improve control performance in the disturbance rejection or tracking quality. Over the past 50 years, considerable attention has been paid to preview control theory. In particular, linear quadratic (LQ) optimal control problems with preview compensation have been studied deeply and some relatively complete theories have been formed [7–15]. A drawback of the LQ optimal design is that it does not consider robustness for the unknown disturbances and system uncertainties.

In order to guarantee robustness for the unknown disturbance, H_2 and H_∞ optimization criteria were introduced into the preview control problems [16–23]. The H_2 preview control problems were discussed for continuous systems with input delay based on an input-delay approach [16,17]. H_∞ preview control problems for a delay system based on the H_∞ control theory were considered in Refs. [18–20] and gain matrices of the preview controller were determined by solving the Riccati equation. The preview control problem in an H_∞ setting was discussed in Refs. [21,22] based on a game theory and the optimal preview controller design method was proposed. The concept of fuzzy control was added to the preview control problem and an H_∞ fuzzy preview controller was presented in the form of LMIs [23].

Similarly, the problems of preview control systems with model uncertainty were extensively discussed in Refs. [24–33]. For norm-bounded uncertain discrete-time systems, the robust H_∞ preview control problem was considered based on the game theoretic approach in Refs. [24,25]. For polytopic uncertain discrete-time systems, the classical difference method in Refs. [11–13] was adopted to derive an augmented error system, and a robust preview controller was designed to achieve robust tracking performance in terms of LMIs in Refs. [26–30]. Based on the robust LQ/H_∞ criterion, a tracking controller with preview actions was designed under the assumption that the reference signal was available in Refs. [31,32]. However, it has been proven that the classical difference methods in Refs. [26–32] based on constant system matrices tend to get conservative results. To overcome the drawbacks of the existing results, the difference between a system state and its steady-state value, instead of the usual difference between system states, was used to derive an augmented error system. This leads to the tracking problem being transformed into a robust H_∞ problem. Therefore, the robust preview controller was obtained by solving an LMI [33].

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Preview control theory has been widely used in engineering practice, such as, vehicle suspension [34], dual-stage actuators [35], and unmanned aerial vehicles [36]. The use of preview control for wind turbine control has also been presented [37].

It should be noted that the existence of desired robust preview controllers proposed by Refs. [16–22,24,25] depended on the positive-semi-definite solution of Riccati equations. As mentioned in Ref. [38], the Riccati-based approach is not universal and is difficult to popularize. A common characteristic of these papers [23,–32] is that they extended the classical difference method directly to uncertain discrete-time systems to construct an augmented error system. Unfortunately, this construction method is not applicable to discrete-time systems with time-varying uncertainties as the uncertainties are often time-varying and unknown, or may be even nonlinear. Therefore, the difference operator cannot be applied for these uncertainties. If the state translation method in Ref. [33] is used, then the structure of the augmented error system will be very complicated, which is not helpful in the design and analysis of preview controllers. In summary, the existed results cannot be applied to general uncertain systems. And the robust preview control problem for uncertain discrete-time systems with time-varying delay based on the error system method has not been considered, to date.

This paper focused on designing a robust preview controller for uncertain discrete-time systems with time-varying delay. A method for construction of the augmented error system for uncertain discrete-time systems with time-varying delay is presented, and the corresponding LMI conditions for designing a robust preview controller are obtained. In comparison with the above-mentioned design methods, the contribution of the proposed solution is that it allows the system matrices to be non-common and to have time-varying uncertainties. In fact, the proposed construction method includes the existing construction methods in Refs. [11–13,23,26–33]. Moreover, the preview controller design based on LMI is simple and easy to popularize.

The paper is organized as follows. The preview control problem and the underlying assumptions are presented in Section 2. Section 3 derives an augmented error system, which includes a discrete integrator and previewable information. In Section 4, the robust preview controller design approach is obtained. A numerical example is given in Section 5 to demonstrate the effectiveness of the proposed approaches. Conclusion is presented in Section 6.

Notations: R^n, $R^{n\times m}$ denote the n-dimensional real vector space and $n\times m$ matrix space, respectively. $P>0$ denotes that the matrix P is positive definite. A^T and A^{-1} represent the transpose and inverse. $\mathrm{G}_1\circ\mathrm{G}_2$ represents the series connection of mapping G_1 and G_2. $\|x\|_2=\sqrt[2]{\sum_{k=0}^{\infty}x(k)^Tx(k)}$ denotes the l_2-norm of x and $\|\cdot\|_\infty$ represents the l_2-induced norm of a transfer function matrix or a general operator. I denotes an identity matrix with appropriate dimensions. In block matrices, we use "*" to denote the terms that can be deduced by symmetry and $diag\{\cdots\}$ stands for a block-diagonal matrix.

2. Problem formulation and basic assumptions

Consider the uncertain discrete-time system with time-varying delay:

$$\begin{cases} x(k+1)=[A+\Delta A]x(k)+[A_d+\Delta A_d]x(k-d(k))+Bu(k), \\ y(k)=Cx(k), \\ x(k)=\varphi(k), k\in[-d_2,0], \end{cases} \tag{1}$$

where $x(k)\in R^n$ is the state vector, $u(k)\in R^q$ is the input control vector, $y(k)\in R^q$ is the output vector, and A, A_d, B and C are known real constant matrices with appropriate dimensions. $\Delta A(k)$ and $\Delta A_d(k)$ are unknown time-varying matrices with appropriate dimensions. $\varphi(k)$ is the given initial condition. $d(k)$ denotes the time-varying delay satisfying

$$\mathrm{d}_1\le d(k)\le d_2, \tag{2}$$

where d_1 and d_2 are the known lower and upper bounds of $d(k)$.

For system (1), the following basic assumptions are made:

A1. $\begin{bmatrix} A+A_d-I & B \\ C & 0 \end{bmatrix}$ is invertible.

A2. There exist real constant matrices $E\in R^{n\times n_1}, H_i\in R^{n_2\times n}$, $(i=1,2)$ and an uncertain matrix $\sum(k)R^{n_1\times n_2}$ such that

$$[\,\Delta A(k)\quad \Delta A_d(k)\,]=E\sum(k)[\,H_1\quad H_2\,], \tag{3}$$

$$\sum(k)^T\sum(k)\le I_{n_2}. \tag{4}$$

Remark 1. A2 is commonly used in robust control. (3) shows that the uncertain matrices of system (1) satisfy matching conditions; (4) shows that ΔA and ΔA_d are time-varying matrices representing norm-bounded parameter uncertainties.

Let $r(k)\in R^q$ be the reference signal, for which we assume that:

A3. The preview length of the reference signal is M_R. This means that at each time k, M_R future values of reference signal $r(k+1)$, $r(k+2)$, $\cdots$, $r(k+M_R)$ as well as the present and past values of the reference signal are available, and the values are assumed to be zero beyond the $k+M_R$, namely

$$r(k+j)=0, j=M_R+1, M_R+2, M_R+3, \cdots \tag{5}$$

Under A3, there exists a constant vector r such that

$$\lim_{k\to\infty} r(k)=r. \tag{6}$$

Remark 2. A3 is the standard assumption of preview control theory. Theoretical research and practical examples have shown that the previewable signal has a significant effect on the performance of the control system only for a certain time period, while there is no noticeable improvement in the system's performance beyond the preview length. Therefore, the future values of the reference signal are assumed to be zero when they exceed the preview length.

As a direct result of the small gain theorem, Lemma 1 is given as follows:

Lemma 1. *(Scaled small gain theorem, SSG)* [39]. *Consider an interconnection system consisting of* S_1 *and* S_2:

$$\begin{aligned} S_1 &: \eta(k)=T(\sigma(k)), \\ S_2 &: \sigma(k)=\Delta(\eta(k)), \end{aligned} \tag{7}$$

where the forward S_1 *is a known linear time-invariant system with operator* T *mapping* $\sigma(k)$ *to* $\eta(k)$, *the feedback* S_2 *is an unknown linear time-varying one with operator* $\Delta\in D=\{\Delta:\|\Delta\|_\infty\le 1\}$ *and* $\eta(k)\in R^\eta$, $\sigma(k)\in R^\sigma$. *Assuming that* S_1 *in* (7) *is internally stable, the closed-loop system of the interconnected system* (7) *is robustly asymptotically stable for all* $\Delta\in D$ *if* $\left\|U_\eta\circ T\circ U_\sigma^{-1}\right\|_\infty<1$ *holds for some*