ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

A novel non-uniform control vector parameterization approach with time grid refinement for flight level tracking optimal control problems

Ping Liu [a,b,1], Guodong Li [c,1], Xinggao Liu [b,*,1], Long Xiao [b], Yalin Wang [d], Chunhua Yang [d], Weihua Gui [d]

[a] Key Lab of Industrial Wireless Network and Networked Control, College of Automation, Chongqing University of Posts and Telecommunications, Chongqing 400065, China
[b] State Key Laboratory of Industry Control Technology, College of Control Science & Engineering, Zhejiang University, Hangzhou 310027, China
[c] China Academy of Electronics and Information Technology, Beijing 100041, China
[d] School of Information Science and Engineering, Central South University, Changsha 410083, China

ARTICLE INFO

Article history:
Received 24 May 2017
Received in revised form
3 November 2017
Accepted 8 December 2017

Keywords:
Flight level tracking
Optimal control
Control vector parameterization
Hilbert-Huang transform
Time grid refinement

ABSTRACT

High quality control method is essential for the implementation of aircraft autopilot system. An optimal control problem model considering the safe aerodynamic envelop is therefore established to improve the control quality of aircraft flight level tracking. A novel non-uniform control vector parameterization (CVP) method with time grid refinement is then proposed for solving the optimal control problem. By introducing the Hilbert-Huang transform (HHT) analysis, an efficient time grid refinement approach is presented and an adaptive time grid is automatically obtained. With this refinement, the proposed method needs fewer optimization parameters to achieve better control quality when compared with uniform refinement CVP method, whereas the computational cost is lower. Two well-known flight level altitude tracking problems and one minimum time cost problem are tested as illustrations and the uniform refinement control vector parameterization method is adopted as the comparative base. Numerical results show that the proposed method achieves better performances in terms of optimization accuracy and computation cost; meanwhile, the control quality is efficiently improved.

1. Introduction

In the last 15–20 years, optimal control methods have received steadily increasing scientific and industrial attention. By calculating the optimal control strategy for a dynamic process, optimal control has been successfully applied in engineering, including ground transportation systems [1,2], air track management systems [3], flight control [4], etc. Because of its importance in application of air track management, flight level optimal control, which plays an important role in autopilot system, has received considerable attentions in recent years [5–8].

Generally, the cruising altitudes of commercial aircraft are typically assigned a flight level by air traffic control (ATC) [6]. To ensure aircraft separation, the flight levels are separated by a few hundred feet. This arrangement is desirable because it will greatly simplify the task of ATC: the problem of ensuring aircraft separation, which is normally three dimensional, can most of the time be decomposed to a number of two dimensional (in some places even one dimensional) problems [9]. However, changes in flight level happen occasionally and must be cleared by ATC. At all other times, the aircraft crew must ensure that they remain within the allowed bounds of their assigned level. At the same time, they must also maintain limits on factors such as speed, flight path angle, and acceleration imposed by limitations of airframe, engine and passenger comfort requirements or to avoid dangerous situations such as aerodynamic stall [7].

To achieve these goals, the aircraft flight level control can be formulated into optimal control problems with constraints and then be solved by using the following three numerical methods: dynamic programming, indirect methods and direct methods [10–13]. Therefore, optimal control methods are essential for the implementation of flight level control. Compared with dynamic programming and indirect methods, direct methods have the advantage that there is no requirement to set up and solve a multipoint boundary value problem associated with Pontryagin's Maxium Principle, the original optimal control problem is directly transformed into a nonlinear programming problem (NLP) by two strategies: complete parameterization (CP) and control vector parameterization (CVP) [14,15]. Consequently, direct methods are more popular for solving the optimal control problems in flight control [4,16].

Since high quality control is essential for the implementation of autopilot system, it is especially important to obtain high quality

* Corresponding author.
E-mail address: lxg@zju.edu.cn (X. Liu).
[1] These authors contributed equally to this work.

https://doi.org/10.1016/j.isatra.2017.12.008
0019-0578/© 2017 ISA. Published by Elsevier Ltd. All rights reserved.

solutions to ensure flight safety and improve the flight stability for the flight level optimal control problems. Considering the dimension of NLP problem in CP method is much greater than that in CVP method and the dynamic system is more accurate [17], this paper mainly focuses on the CVP method. However, the solution quality of CVP method greatly depends on the discretization level (also named time grid), high accuracy solution requires fine discretization grid [18]. A very fine discretization may make the discretized NLP problem very large scale and/or ill-conditioned [18,19], and then increase the computation cost. In addition, it is not possible to select an adequate time grid in advance to the optimization, as the structure of the solution is usually not known a priori. Thus, there exists a challenge to select the optimal time grid to balance the computation cost with the desired solution quality [10,20–23].

To tackle these issues, the Hilbert-Huang transform (HHT), which produces physically meaningful representations of data from nonlinear and non-stationary processes [24,25], is introduced to analyze the nonlinear control variables. By using HHT analysis, the instantaneous frequencies of control variables are obtained so that the relationship between control variables and time grids can be easily analyzed. On this basis, a novel HHT-based time grid refinement approach combined with CVP method is proposed to select optimal time grid nodes so as to obtain high quality solutions for the flight level optimal control problems. In this approach, HHT is adopted to analyze the control profiles obtained in the preceding optimization step and then to refine the time grid nodes by subdividing or eliminating time grid nodes in successive iterations, where the high instantaneous frequency nodes are subdivided to improve the solving accuracy and the low frequency nodes are combined to reduce the computation cost. By introducing the time grid refinement, important nodes are automatically added and unnecessary nodes are combined. Therefore, adaptive time grid nodes are achieved and fewer parameters are needed to calculate high quality control solution and obtain better performance index when compared with the uniform time grid methods. Meanwhile, the computation cost can be efficiently reduced. Finally, two flight level altitude tracking problems and one minimum time cost problem are employed to illustrate the performance of the proposed method in terms of optimization accuracy, control quality and computation cost.

This paper is organized as follows: Section 2 discusses the model of flight level tracking optimal control problem. The piecewise-constant control parameterization method is presented in Section 3. Section 4 shows the HHT-based non-uniform time grid refinement method and Section 5 outlines the implementation of the proposed approach. The numerical tests of flight level optimal control problems are carried out in Section 6. Finally, the conclusion is drawn in Section 7.

2. Flight level tracking optimal control problem

2.1. Aircraft model

The aircraft model commonly used in ATC research [7,9] is adopted, where the movement of the aircraft is restricted in the vertical plane and the motion is described using a point mass model. Meanwhile, a classical model [9], which extends the three dimensions of an aerodynamic envelope protection problem, is directly introduced. Following this model, three coordinate frames are used to describe the motion of aircraft: the ground frame, the body frame and the wind frame. Finally, this three-state model with three state variables $[V \quad \gamma \quad h]$ and two control variables $[T \quad \theta]$ can be briefly described as follows,

$$\begin{bmatrix} \dot{V} \\ \dot{\gamma} \\ \dot{h} \end{bmatrix} = \begin{bmatrix} -a_D V/m - g\sin(\gamma) \\ a_L V(1-c\gamma)/m - g\cos(\gamma)/V \\ V\sin(\gamma) \end{bmatrix} + \begin{bmatrix} 1/m & 0 \\ 0 & a_L V/m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} T \\ \theta \end{bmatrix} \tag{1}$$

where V denotes the speed of the aircraft, γ denotes the flight path angle, h is the altitude, T is the thrust exerted by the engine and θ is the pitch angle, g is the gravitational acceleration, m is the weight of the aircraft, a_L, a_D and c are the corresponding coefficients.

Remark 1. *It can be seen that system (1) has three state variables and is controlled by two inputs, denote* $\boldsymbol{x}(t) := [V \quad \gamma \quad h]$ *and let* $\boldsymbol{u}(t) := [T \quad \theta]$, then, system (1) can be briefly described by a function $\boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t))$ defined as follows,

$$\begin{aligned} \dot{\boldsymbol{x}}(t) &= \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \\ &= \begin{bmatrix} -a_D x_1(t)/m - g\sin(x_2(t)) \\ a_L x_1(t)(1-cx_2(t))/m - g\cos(x_2(t))/x_1(t) \\ x_1(t)\sin(x_2(t)) \end{bmatrix} \\ &\quad + \begin{bmatrix} 1/m & 0 \\ 0 & a_L x_1(t)/m \\ 0 & 0 \end{bmatrix} \begin{bmatrix} u_1(t) \\ u_2(t) \end{bmatrix} \end{aligned} \tag{2}$$

2.2. Flight level optimal control problem

For safety reasons, the aircraft speed and flight path angle must be bounded in a rectangular limitation called "a safe aerodynamic envelop" [7]. Part of the task of the Flight Management System (FMS) is therefore to keep states and controls within safe combinations. On this basis, various control objectives can be considered in the flight level control problems [26], such as flight level tracking, minimum time cost and so on. Therefore, the following Bolza cost function describes these control objectives,

$$g_0(\boldsymbol{u}(t)) = \Phi_0(\boldsymbol{x}(t_f|\boldsymbol{u}(t))) + \int_{t_0}^{t_f} L_0(\boldsymbol{x}(t|\boldsymbol{u}(t)), \boldsymbol{u}(t))dt \tag{3}$$

where $\Phi_0: \mathbf{R}^n \to \mathbf{R}$ and $L_0: \mathbf{R}^n \times \mathbf{R}^r \to \mathbf{R}$ are continuously differentiable functions. Specifically, the flight level optimal control problem then is derived as follows.

Firstly, use the following equations to present the state constraints,

$$\begin{aligned} &g_i(\boldsymbol{x}(t_f), \boldsymbol{u}(t)) = 0, \quad i = 1, 2, \ldots, m_e \\ &g_j(\boldsymbol{x}(t), \boldsymbol{u}(t)) \le 0, \quad j = 1, 2, \ldots, m_{ie} \end{aligned} \tag{4}$$

where m_e and m_{ie} are the numbers of equality terminal constraints and inequality path constraints, respectively; $g_i(\boldsymbol{x}(t_f), \boldsymbol{u}(t))$ and $g_j(\boldsymbol{x}(t), \boldsymbol{u}(t))$ are given continuously differentiable functions. Besides, the control variable bounds are defined as:

$$u_i^L \le u_i(t) \le u_i^U, \quad t \in [t_0, t_f], \quad i = 1, 2 \tag{5}$$

where u_i^L and u_i^U $(i = 1, 2)$ are given real numbers. Let U be the $\boldsymbol{u}(t) \to \mathbf{R}^r$ that satisfies Eq. (5) for all $t \in [t_0, t_f]$. For a given vector $\boldsymbol{u}(t) \in U$, let $\boldsymbol{x}(\cdot|\boldsymbol{u}(t))$ denote the state trajectory of Eq. (2). Let F denote the class of all such feasible controls, where the set of all $\boldsymbol{u}(t) \in U$ satisfies Eq. (4). The aircraft flight level optimal control problem now is stated as follows:

Problem (P1).. *Given the system (2) and the initial condition* $\boldsymbol{x}(t_0) := [x_1(t_0), \ x_2(t_0), \ x_3(t_0)]$, *find a feasible* $\boldsymbol{u}(t) \in F$ such that the cost functional (3) is minimized over F.

Please cite this article as: Liu P, et al. A novel non-uniform control vector parameterization approach with time grid refinement for flight level tracking optimal control problems. ISA Transactions (2017), https://doi.org/10.1016/j.isatra.2017.12.008