ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

# Event-triggered containment control for second-order multi-agent systems with sampled position data

Hong Xia [a,b], Wei Xing Zheng [b,*], Jinliang Shao [a]

[a] School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, PR China
[b] School of Computing, Engineering and Mathematics, Western Sydney University, Sydney, NSW 2751, Australia

ARTICLE INFO

*Article history:*
Received 6 April 2017
Received in revised form
20 October 2017
Accepted 2 November 2017

*Keywords:*
Multi-agent systems
Event-triggered
Sampled position data
Containment control

ABSTRACT

This paper is concerned with the problem of event-triggered containment control for second-order multi-agent systems with sampled position data. First, a distributed event-triggered containment control protocol is designed, which utilizes the sampled position data only and allows the event-triggering condition to be intermittently examined at constant sampling instants. Then, based on the algebraic graph theory and matrix theory, a sufficient condition on the communication topology, the controller gains, and the sampling period is derived so as to achieve containment control. Finally, a numerical example is provided to verify the theoretical results.



## 1. Introduction

The past decade has witnessed dramatic advances on co-operative control in multi-agent systems. As an important and fundamental issue in cooperative control, the consensus problem has received increasing attention from various perspectives [1–10]. Consensus refers to reaching an agreement regarding a certain quantity of interest, and can be classified into leaderless consensus and leader-following consensus, depending upon the absence or presence of a leader. However, in some practical applications, there exist multiple leaders in multi-agent systems. For this case, the leader-following consensus problem becomes a containment control problem, where the control protocol is designed to ensure that the followers will move into the convex hull formed by the leaders. Recently the second-order containment control problem has attracted much attention [11–16]. Cao et al. [11] proposed two containment control algorithms for second-order dynamics in the presence of both stationary and dynamic leaders. The containment control problem was considered in [12] for second-order multi-agent systems with random switching topologies. By utilizing the Lyapunov functional method, Liu et al. [13] addressed the containment control problem for second-order multi-agent systems with time-varying delays, and gave some sufficient conditions to ensure containment control. The work in [14] was devoted to containment control of second-order discrete-time multi-agent systems with Markovian missing data and one step network-induced time delay. The containment control problem was examined for second-order multi-agent systems over a heterogeneous network in [15].

Note that the systems in the above-mentioned literature are with continuous time dynamics. However, in many real situations, such as unreliable information channels and limited data sensing cases, it is not practical to ensure the continuity of the underlying system. In addition, it is more difficult obtain velocity measurements than position measurements. All these facts have led to the study of both consensus and containment control for multi-agent systems with only sampled position measurements [17–21]. A distributed linear consensus protocol with second-order dynamics was designed in [17], where both the current and some sampled past position data are utilized. Since the current information is usually unavailable, both sampled position and velocity data instead of current ones were introduced into the protocol in [18]. Furthermore, by using only causal sampled position data, Huang et al. [19] designed a novel consensus protocol for second-order multi-agent systems. For containment control of multi-agent systems, two distributed algorithms via only position measurements of the agents were developed in [20]. Employing the algebraic graph theory and matrix theory, Zheng et al. [21] considered the formation-containment control problem for second-order multi-agent systems with only sampled position data.

Recently, due to its advantages of improving the utilization rate of resources, event-triggered control has been widely investigated.

* Corresponding author.
*E-mail addresses:* xiahong@uestc.edu.cn (H. Xia), w.zheng@westernsydney.edu.au (W.X. Zheng), jinliangshao@126.com (J. Shao).

https://doi.org/10.1016/j.isatra.2017.11.001
0019-0578/

Instead of depending only on a fixed time period in sampled-data control, the execution of control tasks in event-triggered control is triggered when a specific event occurs, and thus the same control goal can be achieved with less information transmission and controller actuation. Accordingly, event-triggered control has been extended to cooperative control of multi-agent systems, especially in consensus problems and containment control problems [22–25]. In these results, it is necessary for the event-triggering conditions to be continuously monitored, which may result in the requirement of an additional hard device and also is a waste of communication and computation resources. To deal with this circumstance, many researchers have been engaged to integrate event-triggered control with sampled-data control [26–30], since it is more realistic to approximate continuous supervision by measuring the event-triggering condition periodically at constant sampling instants. In [26], an event-triggered strategy and control co-design was proposed for sampled-data control systems to determine whether or not to transmit the sampled data. The problem of leader-following consensus for second-order systems was investigated in [27] in the case when the data are sampled randomly within a certain known bound and the data transmission is driven by an event-triggered control protocol. In [28], Guo et al. studied the event-triggered sampled-data consensus problem for distributed multi-agent systems with directed graph. The event-based consensus problem of general linear multi-agent systems was considered in [29], and two sufficient conditions to guarantee the consensus were presented therein for cases with or without continuous communication between neighboring agents. According to the Lyapunov stability theory and graph theory, Hu et al. [30] developed some event-triggered protocols depending on sampled-data information for the cluster consensus problem.

It should be pointed out that information of both the position and the velocity states are required in the aforementioned event-triggered control for second-order multi-agents systems. However, as has been shown before, the velocity measurements of agents are often unavailable in many real-word applications. Therefore, we are motivated to design a new event-triggered protocol with only sampled position data for second-order multi-agent system. On the other hand, the existing results on event-triggered strategies have mostly adopted Lyapunov-type argument. It is well known that Lyapunov function is often difficult to select. Moreover, the simple quadratic Lyapunov function is usually no longer suitable in the case of time-dependent threshold. This inspires us to make the convergence analysis based on the algebraic graph theory and matrix theory.

In this paper, we consider the event-triggered containment control problem for second-order multi-agent systems with sampled position data. First of all, we propose a distributed event-triggered containment control protocol, which involves the sampled position data only, and requires the event-triggering condition to be periodically instead of continuously measured and calculated. Secondly, a sufficient condition (which depends upon the interaction graph, the control gains and the sampling period) is established to guarantee containment control. It is shown that the followers are driven into the convex hull spanned by the dynamic leaders, if for each follower there exists at least one leader that has a directed path to the follower, the sampling period is upper bounded by several positive values and the control gains in the protocol are properly chosen. Compared with the most of the existing works, the convergence analysis is based on the graph theory and matrix theory rather than the Lyapunov-based stability theory, and an explicit formulation for the tolerance interval of the sampling period is also given.

The rest of this paper is organized as follows. Section 2 introduces some mathematical preliminaries and formulates the problem considered in this paper. The main results are presented in Section 3. In Section 4, some simulation results are given to show the effectiveness of the proposed method. Finally, Section 5 concludes the paper.

## 2. Preliminaries and formulation

For an $n$-agent system, the interaction topology among agents can be modeled by a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_n\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ denotes, respectively, the vertex set and the edge set. An edge in $\mathcal{E}$ is represented by an ordered pair $(v_j, v_i)$, where $(v_j, v_i) \in \mathcal{E}$ if agent $i$ can access the state information of agent $j$ but not necessarily vice versa. The index set of neighbors of vertex $v_i$ is denoted by $\mathcal{N}_i = \left\{ v_j \in \mathcal{V} | (v_j, v_i) \in \mathcal{E}, j \neq i \right\}$. An agent is called a leader if it has no neighbors, and is called a follower if it has at least a neighbor. A directed path is a sequence of edges in a directed graph of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \cdots$, where $v_{i_j} \in \mathcal{V}$. Analogously, an undirected path can be defined. A nonnegative matrix $A = (a_{ij}) \in \mathbb{R}^{n\times n}$ can be associated with a graph $\mathcal{G}(A)$ in such a way that $A$ is specified as the weighted adjacency matrix. In detail, for $v_i, v_j \in \mathcal{V}$, $v_j \in \mathcal{N}_i \Longleftrightarrow a_{ij} > 0$. The Laplacian matrix $L = (l_{ij}) \in \mathbb{R}^{n\times n}$ associated with $A$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{n} a_{ij}$ and $l_{ij} = -a_{ij}$.

In this paper, we suppose that there are $m$ leaders labelled as agents 1 to $m$, and $n - m$ followers labelled as agents $m + 1$ to $n$. For simplicity, the set of leaders and the set of followers are denoted by $\mathcal{R}$ and $\mathcal{F}$, respectively. According to the definitions of leaders and followers, $L$ can be partitioned as

$$\begin{pmatrix} L_1 & L_2 \\ \mathbf{0}_{m\times(n-m)} & \mathbf{0}_{m\times m} \end{pmatrix}, \tag{1}$$

where $L_1 \in \mathbb{R}^{(n-m)\times(n-m)}$ and $L_2 \in \mathbb{R}^{(n-m)\times m}$.

Let $I$ denote an identity matrix with a compatible dimension. $\rho(A)$ and $\|A\|$ represent the spectral radius of matrix $A$ and the matrix norm of $A$, respectively. $\otimes$ stands for the Kronecker product. $\lfloor x \rfloor$ refers to the floor function, which means the largest integer less than or equal to $x$. Let $\prod_{k=v}^{s} A(k)$ denote the successive matrix product from $s$ to $v$, that is, $\prod_{k=v}^{s} A(k) = A(s)A(s-1)\cdots A(v)$.

**Assmption 1.** The communication topology between leaders and followers is directed, while the communication topology among followers is undirected.

**Assmption 2.** For each follower, there exists at least one leader that has a directed path to the follower.

**Lemma 1** ([31]). *Under Assumption 2, all the eigenvalues of $L_1$ are positive, each element of $-L_1^{-1}L_2$ is nonnegative, and the sum of each row of $-L_1^{-1}L_2$ is 1.*

**Lemma 2** ([32]). *For a third-order real coefficient polynomial $f(s) = a_3s^3 + a_2s^2 + a_1s + a_0$, $f(s)$ is stable if and only if $a_3, a_2, a_1, a_0$ are positive and $a_1a_2 - a_0a_3 > 0$.*

**Lemma 3** ([33]). *Let $A \in \mathbb{R}^{n\times n}$ and $\varepsilon > 0$. There is a matrix norm $\|\cdot\|$ such that $\rho(A) \le \| A \| \le \rho(A) + \varepsilon$.*

**Lemma 4** ([34]). *(Stolz-Cesaro Theorem)If $\{b_k\}_{k=1}^{\infty}$ is a sequence of positive real numbers such that $\sum_{k=1}^{\infty} b_k = \infty$, then for any sequence $\{a_k\}_{k=1}^{\infty} \subset \mathbb{R}$ the following inequalities hold:*

$$\limsup_{k\to\infty} \frac{a_1 + a_2 + \cdots + a_k}{b_1 + b_2 + \cdots + b_k} \le \limsup_{k\to\infty} \frac{a_k}{b_k},$$
$$\liminf_{k\to\infty} \frac{a_1 + a_2 + \cdots + a_k}{b_1 + b_2 + \cdots + b_k} \ge \liminf_{k\to\infty} \frac{a_k}{b_k}.$$

*In particular, if the sequence $\{a_k/b_k\}_{k=1}^{\infty}$ has a limit, then*

Please cite this article as: Xia H, et al. Event-triggered containment control for second-order multi-agent systems with sampled position data. ISA Transactions (2017), https://doi.org/10.1016/j.isatra.2017.11.001