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Practice article

Robust synchronization of master-slave chaotic systems using approximate model: An experimental study

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ABSTRACT

Robust synchronization of master slave chaotic systems are considered in this work. First an approximate model of the error system is obtained using the ultra-local model concept. Then a Continuous Singular Terminal Sliding-Mode (CSTSM) Controller is designed for the purpose of synchronization. The proposed approach is output feedback-based and uses fixed-time higher order sliding-mode (HOSM) differentiator for state estimation. Numerical simulation and experimental results are given to show the effectiveness of the proposed technique.

1. Introduction

Over the last decades, the synchronization of chaotic systems has attracted a lot of attention of researchers from multidisciplinary research communities [1,2]. Synchronization of chaotic systems has several potential applications. It can be used for secure communication [3,4], electronic locking device [5], chemical and biological systems [6], neural network [7], signal processing [8] etc.

In the context of chaotic system synchronization, one important problem is the synchronization of master-slave systems. A collection of master-slave chaotic systems can be consulted from Ref. [9]. In this problem, the slave system needs to follow the trajectory of the master system. This problem has been studied very widely in the literature and as such various results are available. Various control approaches have been applied or can be applied for master-slave synchronization of chaotic systems, for example, linear output-feedback [10], linear state-feedback [11], state feedback linearization [12], sliding-mode [13], adaptive sliding-mode [14], proportional-derivative controller [15], nonlinear $\mathcal{H}_\infty$ controller [16], active disturbance rejection control [17] etc. Most of these controllers provide relatively good performances.

However, in order to design the previously mentioned robust controllers, good knowledge of the system dynamics are required. Obtaining good models are often not so easy due to various practical considerations like uncertainties, and external disturbances. To overcome the limitation of model based control, an alternative solution can be to use approximate model based control [18,19]. The main idea here is to approximate the system dynamics by a local model described by an appropriate input-output relationship.

Based on the previously mentioned works, this article proposes an approximate model based sliding-mode control strategy to achieve robust synchronization of master-slave chaotic systems. The novelty of this paper is to combine approximate-model and sliding-mode control techniques in order to control uncertain nonlinear systems like chaotic oscillators. Moreover, experimental verification is provided as well.

Main contributions: Firstly, unlike the existing model based approaches available in the literature e.g. [15], an approximate-model based master-slave synchronization is proposed here. This is a big advantage over the existing results. Secondly, the dimension of the observer being used for synchronization is lower than the disturbance estimation based control schemes [20]. Finally, experimental validation is another contribution of this work.

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In this work, the synchronization error system is approximated by an ultra-local model. Using this approximate model, a sliding-mode controller is designed so that the slave systems can track the master system. The proposed controller is output-feedback based and uses fixed-time Higher Order Sliding-Mode (HOSM) differentiator (see [21], for an introduction of HOSM differentiator and [22–25] for various applications) to estimate the states and perturbations. The proposed controller produces less chattering than the conventional sliding-mode controller. Moreover it does not require the estimate of the disturbance like [17,20].

The rest of the article is organized as follows: Problem statement is given in Section 2, details of the proposed control strategy can be found in Section 3. Simulation results are given in Section 4 while experimental study can be found in Section 5. Finally Section 6 concludes this article.

2. Problem statement

Consider the following master system

$$\mathbf{\Sigma}_M : \begin{cases} \dot{x}_M &= f_M\left(x_M\right) \\ y_M &= h_M\left(x_M\right) \end{cases} \tag{1}$$

where $x_M = [x_{1M} \quad x_{2M} \quad \dots \quad x_{nM}]^T \in \mathbb{R}^n$ is the state vector and $y_M \in \mathbb{R}$ is the output of the master system. f_M and h_M are smooth vector fields. Consider a slave system described by

$$\mathbf{\Sigma}_S : \begin{cases} \dot{x}_S &= f_S\left(x_S\right) + g_S\left(x_S\right) u \\ y_S &= h_S\left(x_S\right) \end{cases} \tag{2}$$

where $x_S = [x_{1S} \quad x_{2S} \quad \dots \quad x_{nS}]^T \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ ($u\colon \mathbb{R}_+ \to \mathbb{R}$ is locally essentially bounded and measurable signal) is the control and $y_S \in \mathbb{R}$ is the output of the slave system. f_S, g_S and h_S are smooth vector fields. Now, the synchronization problem considered can be established as follows.

The master-slave synchronization objective: Given two chaotic system of same order, find a control to force the states of the slave system (2) to be synchronized with the states of the master system (1). In order to attain this objective, let us define the synchronization error, $\varepsilon := x_M - x_S$. Then, master-slave synchronization is defined as follows:

Definition 1. A slave system (2) exhibits master-slave synchronization with the master system (1), if

$$\lim_{t\to\infty} \varepsilon = 0 \tag{3}$$

for all $t \geq 0$ and any initial condition $\varepsilon(t_0) = x_M(t_0) - x_S(t_0)$.

3. Approximate model based sliding-mode control

This section provides the detail of the proposed controller which will be used later for the purpose of master-slave synchronization. Consider a general nonlinear Single-input Single-Output (SISO) chaotic system represented in the general form as [26].

$$f\left(y, \dot{y}, \dots, y^{(a)}, u, \dot{u}, \dots, u^{(b)}, d\right) = 0, \tag{4}$$

where $y \in \mathbb{R}$ is the measurable output, $u \in \mathbb{R}$ is the input signal and $d \in \mathbb{R}$ is a bounded disturbance.

Property 1. *The system (4) is BIBS (bounded input bounded state), and the derivative of its input is bounded.*

Property 1 is not restrictive as it can be fulfilled by chaotic systems. It is not possible to have chaotic behavior without boundedness of the trajectory. However, it is to be noted here that for general nonlinear systems, Property 1 can be restrictive.

Roughly speaking, the main idea of approximate model based control is to replace complex "unknown" mathematical model by a simple ultra-local model which is only valid during a very short time interval. In this direction, model (4) can be approximated by the following ultra-local model

$$y^{(\nu)} = F + \alpha u, \tag{5}$$

where ν is the derivative of order $\nu \geq 1$ of y, F is the compensation term, which carries the unknown and/or nonlinear dynamics of the system as well as the time-varying external disturbances and $\alpha \in \mathbb{R}$ is a "nonphysical" constant parameter for scaling. The compensation term F can be estimated by the measurements of the system input and output.

Assumption 2. The disturbance signal $F : \mathbb{R}_+ \to \mathbb{R}$ is continuously differentiable for almost all $t \geq 0$, and there is a constant $0 < \kappa^+ < \infty$ such that $ess\ sup_{t\geq 0}|\dot{F}(t)| \leq \kappa^+$.

The existing literature on the application of model-free control depends on the estimation of F (e.g., [18,19]). In this work we will not consider this direction. In our case, F will be considered as a disturbance and the objective is to design a robust controller in the presence of F using differentiator based approach. In this paper, uniform finite-time convergent differentiator proposed by Cruz-Zavala et al. [27] will be used for the purpose of state estimation. This will be detailed later on in this Section. To design the tracking controller, the tracking errors can be defined as $e_1 = y - y_d$ and $\dot{e}_1 = e_2 = \dot{y} - \dot{y}_d$. Then the tracking error dynamics can be written as

$$\dot{e}_1 = e_2, \tag{6a}$$

$$\dot{e}_2 = F + \alpha u - \ddot{y}_d. \tag{6b}$$

The tracking problem for system (5) is essentially the stabilization of the error dynamical system (6). To stabilize system (6), it is necessary to design a controller u under the presence of external perturbations and/or parametric uncertainties which are included in the unknown function F. For this purpose, inspired by the ideas given in Refs. [28,29], the following CSTSM controller is used in this work:

$$\sigma_L\left(e_1, e_2\right) = e_1 + \frac{\alpha}{L^{0.5}}\lceil e_2 \rfloor^{\frac{2}{3}}, \tag{7a}$$

$$u = \ddot{y}_d - k_1 L^{\frac{2}{3}} \lceil \sigma_L \rfloor^{\frac{1}{3}} + z, \tag{7b}$$

$$\dot{z} = -k_2 L \lceil \sigma_L \rfloor^0, \tag{7c}$$

where $\sigma_L\left(e_1, e_2\right)$ is a continuously differentiable function of the state, $\ddot{y}_d$ is the second derivative of the reference trajectory to be tracked, z is a dummy variable which extends the dynamics of the system so that the control signal become continuous, k_1, k_2 and L are design parameters of the proposed control law and $\alpha > 0$ is a constant. Controller (7) requires all components of the state vector, which limits its implementation. However, in order to overcome this difficulty, observers/differentiators are proposed for solving the mentioned problem about state estimation.

3.1. Fixed-time differentiator for state estimation

According to Property 1, system (4) is BIBS. Model (5) is a local approximation of model (4). So, similar assumptions are also applicable in this case. Lets consider that the upper bound for F is f^+, i.e. $|\frac{dF}{dt}| \leq f^+$. To estimate the states of Model (5), the following HOSM differentiator can be applied based on [27]: