ISA Transactions ∎ (∎∎∎∎) ∎∎∎–∎∎∎

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

Two stage neural network modelling for robust model predictive control

Krzysztof Patan

Institute of Control and Computation Engineering, University of Zielona Góra, ul. Szafrana 2, 65-516 Zielona Góra, Poland

ARTICLE INFO

Article history:
Received 14 April 2017
Received in revised form
17 August 2017
Accepted 19 October 2017

Keywords:
Predictive control
Robustness
Neural networks
Stability

ABSTRACT

The paper proposes a novel robust model predictive control scheme realized by means of artificial neural networks. The neural networks are used twofold: to design the so-called fundamental model of a plant and to catch uncertainty associated with the plant model. In order to simplify the optimization process carried out within the framework of predictive control an instantaneous linearization is applied which renders it possible to define the optimization problem in the form of constrained quadratic programming. Stability of the proposed control system is also investigated by showing that a cost function is monotonically decreasing with respect to time. Derived robust model predictive control is tested and validated on the example of a pneumatic servomechanism working at different operating regimes.

1. Introduction

Since the first recurrent neural networks models were developed in the early 1980s recurrent networks proved their usefulness in the control area in both modelling [8,24,27] and control [15,25] of nonlinear dynamic processes. Artificial neural networks provide an excellent mathematical tool for dealing with non-linear problems. They have an important property according to which any continuous non-linear relationship can be approximated with arbitrary accuracy using a neural network with a suitable architecture and weight parameters [8,7]. Furthermore, a neural network can extract the system features from historical training data using the learning algorithm, requiring little or no *a priori* knowledge about the system. This provides the modelling of nonlinear systems with a great flexibility, especially in cases where there is no possibility to find out the analytical input-output representation of the system.

Nowadays Model Predictive Control (MPC) is a popular and frequently used control scheme which succeeded in industrial applications [16,34,9,12,23]. However, robustness of MPC against model uncertainty and noise is still a challenge. Robustness of a control system is referenced to a specific uncertainty range and specific stability and performance criteria. In spite of a rich literature devoted to robust control of linear systems, further research is required to develop implementable robust controllers, especially for nonlinear plants [21].

Generally, in the field of robustness the model uncertainty follows from two main sources [1]: (i) unmodelled dynamics of a plant, (ii) unmeasured noise/disturbances which enter the plant. In the framework of linear time-invariant systems different approaches have been proposed, e.g. impulse/step responses, a polytopic uncertainty or bounded input disturbances. Generally speaking, the existing methods can be divided into two classes: structured and unstructured uncertainties [33]. These uncertainty descriptions, however, are very useful in the case of linear time-invariant systems, especially using H_∞ paradigm [6]. An intuitive method for achieving the robustness is to solve min-max problem [14]. However, such an approach leads to very time consuming algorithms; problems are especially observed in the case of nonlinear problems. A remedy for this it could be the application of approximate reachable sets determined by means of the interval arithmetic [18]. A drawback of this solution is that constraints are formulated using uncertain evolution sets which can complicate the optimization process. Recently, neural networks has been applied to robust MPC synthesis [17,37]. In these works, however, recurrent neural networks are used to solve an optimization problem. Studies on incorporating neural networks for the robust model developing in the context of MPC are rather scarce.

This paper tries to fill this gap. It is proposed to use dynamic neural networks twofold: to derive the nominal model of a plant and to deal with the uncertainty associated with the nominal model. Such an approach is motivated by the fact that in many cases a controlled plant is poorly damped or has small stability margin and data representing the plant has to be recorded in the closed loop control with a preliminary selected controller. Consequently, the quality of the model will be limited to the properties of the controller used to record the data. Then, the model should be improved in some way. The problem of identifying dynamic systems in two steps in not new. [22] proposed Set Membership identification for achieve models of good performance and robustness. They assumed that the identification error is unknown

E-mail address: k.patan@issi.uz.zgora.pl

http://dx.doi.org/10.1016/j.isatra.2017.10.011

but bounded. In their work they used the preliminary estimate of the system and then derived a correction model which may give accuracy improvement. In turn [31] developed a scheme called Model Error Modeling (MEM) where prediction error methods are employed to identify a model from the input-output data. Then the uncertainty of the model can be derived by analyzing residuals evaluated from the inputs. This provides the so-called *error model*. In the original algorithm, a nominal model along with uncertainty is constructed in the frequency domain adding frequency by frequency the model error to the nominal model [31]. The concept of MEM was extended to the time domain and nonlinear case and investigated in the framework of robust fault diagnosis by [27]. In turn the synthesis of robust MPC based on neural networks and MEM was signaled in the work of [29].

The paper presents a novel procedure for robust MPC synthesis in which a neural network is used to estimate uncertainty associated with the plant model. The proposed data-driven method to derive uncertainty is an interesting alternative to classical methods such as structural or parametric uncertainty which require uncertainty to be formulated in the analytical way. The proposed solution uses data easily available in the control system. From that point of view the developed solution is the superiour over the structural or parametric ones, especially when nonlinear processes are considered. The contributions of the paper are as follows.

1. To develop two-stage modelling procedure aimed in achieving a robust model of a plant. For this purpose two neural network models are used.
2. To design robust constrained MPC based on instantaneous linearization of a robust model. Thus, the optimization carried out in the framework of MPC is simply turned out to quadratic programming.
3. To provide stability conditions for the robust MPC by means of terminal constraints.
4. To carry out experiments illustrating the efficacy of the approach considered.

The paper is organized as follows. After Introduction, in Section 7.1, system modelling and uncertainty handling are described. Then, in Section 3 nonlinear model predictive control based on a neural network predictor is portrayed. The subsequent section deals with the synthesis of a robust controller. In Section 5 the robust stability of the proposed control scheme is presented and proved. The idea of performance testing of the control scheme is depicted in Section 6. Experimental results are shown and discussed in Section 7. The last section contains conclusions and final remarks.

2. System modelling

Let consider a nonlinear system represented as:

$$y(k+1) = f(\boldsymbol{\varphi}(k)), \tag{1}$$

where f is a nonlinear function, $\boldsymbol{\varphi}(k)$ is a regression vector of the form $\boldsymbol{\varphi}(k) = [y(k), \dots, y(k-n_a+1), u(k), \dots, u(k-n_b+1)]^{\mathrm{T}}$, T stands for the transposition operator, n_a and n_b represent the number of past outputs and inputs needed for designing the model, respectively. It should be kept in mind that f, $y(k)$ and $\boldsymbol{\varphi}(k)$ are not exactly known, however a set of noise corrupted measurements $\tilde{y}(k)$ and $\tilde{\boldsymbol{\varphi}}(k)$ of $y(k)$ and $\boldsymbol{\varphi}(k)$, respectively, is available then

$$\tilde{y}(k+1) = f(\tilde{\boldsymbol{\varphi}}(k)) + \varepsilon(k), \tag{2}$$

where $\varepsilon(k)$ stands for the additive white measurement noise. Our goal is to find an estimate $\hat{f}$ of the nonlinear function f in such a way as to achieve possibly minimal modelling error $|f - \hat{f}|$. This estimate is usually obtained by a prediction error method. Among many approaches reported in the literature artificial neural networks can be successfully applied [24,8]. In the field of neural modelling the simplest solution is to use the feedforward networks with external dynamics [27,25]. There are two strong arguments for using this structure. First of all, such a topology is able to approximate any nonlinear mapping with arbitrary accuracy [11]. The second one is that such a structure is the most popular among the control community as it is very simple to both train and implement. In this paper, the neural model with one hidden layer is considered [8]:

$$\tilde{y}(k+1) = \hat{f}(\tilde{\boldsymbol{\varphi}}(k)) = \sigma_o(\boldsymbol{W}_2^{\mathrm{T}}\sigma_h(\boldsymbol{W}_1^{\mathrm{T}}\tilde{\boldsymbol{\varphi}}(k) + \boldsymbol{b}_1) + \boldsymbol{b}_2), \tag{3}$$

where $\boldsymbol{W}_1 \in \mathbb{R}^{(n_a+n_b)\times v}$ and $\boldsymbol{W}_2 \in \mathbb{R}^{v\times 1}$ are weight matrices of hidden and output layers, respectively, $\boldsymbol{b}_1 \in \mathbb{R}^v$ and $\boldsymbol{b}_2 \in \mathbb{R}^1$ are bias vectors of hidden and output units, respectively, $\sigma_h: \mathbb{R}^v \to \mathbb{R}^v$ is the activation function of the hidden layer, and $\sigma_o: \mathbb{R}^1 \to \mathbb{R}^1$ is the activation function of the output layer and v stands for the number of hidden neurons. Weight matrices and bias vectors are subject of training which is usually carried out using data recorded in the plant working at the normal operating conditions either in the open or closed loop control according as conditioning.

2.1. Uncertainty handling

A control system is robust if it is insensitive to differences observed between the plant and the model used for controller synthesis. Every modelling procedure either for linear or nonlinear processes suffers from the so-called the model mismatch, c.f. the model of the system is not a faithful replica of plant dynamics. On this basis, the uncertainty can be seen as a measure of unmodelled dynamics, noise and disturbances affecting the plant [30]. Moreover, it should be noted that if a model is derived in the closed loop control (with a preliminary selected controller), its quality is limited to the properties of the controller used to record the data. When such a model is used to design MPC the quality of the control may be questionable. Consequently, a model should be improved in some way. This section proposes an approach to deal with these aspects.

Let consider the following uncertain nonlinear discrete-time system

$$y(k) = f(\boldsymbol{\varphi}(k)) + w(k), \tag{4}$$

where $w(k)$ represents the uncertainty which is assumed to be additive and bounded in a compact set $\mathcal{W}$ ($w(k) \in \mathcal{W}$). This kind of uncertainties if often called global uncertainties [3]. At the first view global uncertainty seems to be more disturbances than uncertainty because it influences the system in the similar way as external perturbations. However, $w(t)$ may be expressed as a function of past system inputs and outputs and (4) can be rewritten as follows:

$$y(k) = f(\boldsymbol{\varphi}(k)) + \Delta f(\boldsymbol{\varphi}(k)) = \bar{f}(\boldsymbol{\varphi}(k)). \tag{5}$$

Then, the additive uncertainty can represent a wide class of model mismatches [18].

Starting from (4) and using the nominal model of the plant as well as available measurements, the uncertainty can be estimated as:

$$w(k) = y(k) - f(\boldsymbol{\varphi}(k)) \approx \tilde{y}(k) - \hat{f}(\boldsymbol{\varphi}(k)) = \tilde{y}(k) - \hat{y}(k). \tag{6}$$

The Eq. 6 looks like a residual, cf. a signal defined as a difference between the measured process output and the output of a model,

Please cite this article as: Patan K. Two stage neural network modelling for robust model predictive control. ISA Transactions (2017), http://dx.doi.org/10.1016/j.isatra.2017.10.011