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Robust cubature Kalman filter for GNSS/INS with missing observations and colored measurement noise

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ABSTRACT

In order to improve the accuracy of GNSS/INS working in GNSS-denied environment, a robust cubature Kalman filter (RCKF) is developed by considering colored measurement noise and missing observations. First, an improved cubature Kalman filter (CKF) is derived by considering colored measurement noise, where the time-differencing approach is applied to yield new observations. Then, after analyzing the disadvantages of existing methods, the measurement augment in processing colored noise is translated into processing the uncertainties of CKF, and new sigma point update framework is utilized to account for the bounded model uncertainties. By reusing the diffused sigma points and approximation residual in the prediction stage of CKF, the RCKF is developed and its error performance is analyzed theoretically. Results of numerical experiment and field test reveal that RCKF is more robust than CKF and extended Kalman filter (EKF), and compared with EKF, the heading error of land vehicle is reduced by about 72.4%.

1. Introduction

In the research of integrating global navigation satellite system (GNSS) with inertial navigation system (INS), data fusion is a most active research area, where Kalman filter (KF) and its variants have been widely applied. Due to the inherent disadvantages of wireless communication, GNSS suffers from signal attenuation, dense multipath and signal outages inevitably, which degrades the performance of KFs. Besides adding extra sensors, which may increase the cost and bring new problems in data fusion, improving the data fusion methods can help in mitigating the drift error of INS during GNSS-denied duration [1,2]. There is no satisfactory result from extended Kalman filter (EKF) when large system uncertainties occur, i.e., EKF may diverge quickly when short signal outages appear. With the development of computer hardware, many artificial intelligence (AI) methods have been introduced into the state estimation of GNSS/INS [3,4], which generally require a training period to adjust their model parameters. Because of high noise of sensors or frequent short signal outages in urban environment, AI-based methods are time-consuming or less efficient in real-time application [5]. It has been reported that when the duration of signal outage is short (e.g., less than 60 s) KFs outperforms AI-based methods [1], so we focus on improving nonlinear filter to handle the frequent signal outages problem.

When the navigation system suffers from signal outages and dense multipath, the measurement noise becomes time-correlated and non-Gaussian [6], making the innovations of KFs larger and the filter unstable in the sequel. It is generally recognized that particle filter (PF) has superiority in handling nonlinear and non-Gaussian problems [7]. Boucher et al. utilized a hybrid filter based on PF and KF to bridge the GPS outages [8], which however is time-demanding in case the involved system dimension is high. Due to their positive features in better accuracy than EKF and less computational cost than PF, sigma-point KF (SPKF) has drawn great attention in recent decade, such as unscented KF (UKF) [9], Gauss-Hermite quadrature filter [10] and more recently cubature KF (CKF) [11]. Unlike PF where the sigma points are randomly generated, SPKF employs deterministic sampling that needs much less sigma points than PF. Many works have been done to verify the superiority of SPKF in integrated navigation [12,13], the results of which indicate that SPKF outperforms EKF in terms of convergence rate and robustness towards system uncertainties. Xu et al. have verified that CKF outperforms UKF for estimation problem with high state dimension [14], as the general dimension of GNSS/INS is 15 or higher, we focus on the usage of CKF in our work.

It has been verified that the nonlinearity approximation error of EKF for the measurement function of tight GNSS/INS can be neglected [15], but the comparison in [16] revealed that EKF is more easily to be degraded by external disturbances than UKF. In other

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words, the approximation error is not the main factor that leads to the failure of EKF in GNSS/INS, and invalid assumption of standard KF is the major cause of performance degradation. Many robust filtering methods have been proposed to solve the problems of KFs, i.e., the invalid Gaussian assumption and inevitable model uncertainties. Masreliez et al. proposed a robust KF by designing a nonlinear *score function* to deemphasize the large innovations [17], which however, needs a prior knowledge to fix the *score function*. Parameters identification of scarce measurement systems has been widely reported [18–20], Wang and Ding proposed gradient-based and least squares-based method to identify the unknown parameters of system model [21,22], where a batch process of the observations are involved, making their usage in real-time navigation needs further verification. Shmaliy proposed a robust Kalman-like FIR estimator to remove the dependence of KFs on initial state errors and noise statistics [23], Shi et al. studied the stochastic incomplete measurement by using H_∞ filtering [24], both of which show better performance than KFs when large system uncertainties are involved. However, the selection of averaging horizon length has a great effect on the performance of robust FIR estimator [25], and the H_∞ filter would make a tradeoff between filtering robustness and accuracy.

It has been reported that the assumption of a Gaussian state predictive density at each step is closely satisfied in many practical applications [17]. In the calculation of posterior probability density function (PDF), the predicted sigma points should be weighted more than the observations with non-Gaussian and colored noise. A novel sigma point update method without constructing the prior PDF was proposed by Tian [26], and García-Fernández pointed out that the Bayesian update framework is less efficient when the noise of observations is small [27]. Inspired by the work of Tian and García-Fernández, an efficient Bayesian update framework is developed in this paper to enhance the robustness of CKF in case frequent signal outages occur. More specifically, the odd and higher order terms of the covariance Taylor series are retained in the approximation of the likelihood, which makes the approximation of posterior PDF more accurate. The main contributions of this paper can be summarized as follows: (1) it is the posterior sigma points error not the first two moments of PDF that are reused in the next filtering period of CKF, which provides more prior information than traditional re-sampling method, (2) the uncertainties from colored noises are translated into the generation of sigma points, where no explicit parameters are needed, (3) the proposed robust CKF is suitable for online integrated navigation.

The rest of this paper is arranged as follows. In Section 2, an improved cubature Kalman filter is derived by considering colored measurement noise. In Section 3, the novel sigma point update framework is presented, based on which a new filter is developed and its error performance is analyzed. The result of field test is reported in Section 4. Finally, some conclusions are drawn based on the simulation results.

2. Cubature Kalman filter with colored measurement noise

Consider a discrete nonlinear system

$$x_k = f(x_{k-1}) + \omega_{k-1} \tag{1}$$

$$z_k = h(x_k) + \nu_k \tag{2}$$

where $x_k \in \Re^{n\times 1}$, $z_k \in \Re^{p\times 1}$ are the state and measurement vector at time t_k, ω_{k-1} is Gaussian white noise and satisfies $\omega_{k-1}\sim N(0, Q_{k-1})$, ν_k is colored measurement noise and satisfies

$$\nu_k = \psi_k \nu_{k-1} + \varsigma_k \tag{3}$$

where $\psi_k = \exp(-\kappa T)$, κ is time constant and T is the sample interval, ς_k is Gaussian white noise and satisfies $\varsigma_k\sim N(0, R_k)$. The initial state x_0, ω_{k-1} and ν_k are mutually independent, $x_0\sim N(\hat{x}_0, P_0)$.

Let $x_{k-1}\sim N(\hat{x}_{k-1|k-1}, P_{k-1|k-1})$ be the posterior state at time t_{k-1}, $x_k\sim N(\hat{x}_{k|k-1}, P_{k|k-1})$ be the predicted state at time t_k. $m = 2n$ and $w_i = 1/m$ the algorithm of original CKF (with $\psi_k = 0$) is presented in Algorithm 1. Now consider the observation z_k with colored noise

$$z_k = \beta_k H_k x_k + \nu_k \tag{4}$$

where $H_k = \partial h(x_k)/\partial x_k |_{x_k=\hat{x}_{k|k-1}}$ and $\beta_k = diag(\beta_{1,k}, \beta_{2,k}, \cdots, \beta_{p,k})$ is used to take the nonlinear approximation residuals into account. Apply the time-differencing approach from [6] to yield new measurement equation as

$$\begin{aligned} z_k^* &= \beta_k H_k x_k + \nu_k - \psi_k(\beta_{k-1}H_{k-1}x_{k-1} + \nu_{k-1}) \\ &= \beta_k H_k x_k + \varsigma_{k-1} - \psi_k \beta_{k-1} H_{k-1} x_{k-1} \end{aligned} \tag{5}$$

In order to simplify (5), let the system function be formulated as

$$x_k = \alpha_k F_k x_{k-1} + \omega_{k-1} \tag{6}$$

where $F_k = \partial f(x_k)/\partial x_k|_{x_k=\hat{x}_{k-1|k-1}}$, $\alpha_k = diag(\alpha_{1,k}, \alpha_{2,k}, \cdots, \alpha_{n,k})$. Then we get

$$x_{k-1} = (\alpha_k F_k)^{-1}(x_k - \omega_{k-1}) \tag{7}$$

Let $B_k = \beta_k H_k$, $A_k = \alpha_k F_k$, substituting (7) into (5) we have

$$z_k^* = \underbrace{\left[B_k - \psi_k B_{k-1} A_k^{-1}\right]}_{H_k^*} x_k + \underbrace{\psi_k B_{k-1} A_k^{-1} \omega_{k-1} + \varsigma_k}_{\nu_k^*} \tag{8}$$

Because ς_{k-1}, ω_{k-1} are uncorrelated white noises, so ν_k^* is Gaussian white noise. However, as ν_k^* is an explicit function of ω_{k-1}, the process noise and measurement noise are correlated according to

$$\begin{aligned} C_k &= E\left[\omega_{k-1}\left(\nu_k^*\right)^T\right] \\ &= E\left[\omega_{k-1}\left(\psi_k B_{k-1} A_k^{-1} \omega_{k-1} + \varsigma_k\right)^T\right] \\ &= Q_{k-1}\left(A_k^{-1}\right)^T B_{k-1}^T \psi_k^T \end{aligned} \tag{9}$$

and the gain of the filter and the posterior covariance can be written as [28]

$$K_k = \left(P_{k|k-1}\left(H_k^*\right)^T + C_k\right) \left[H_k^* P_{k|k-1}\left(H_k^*\right)^T + R_k^* + H_k^* C_k + C_k^T\left(H_k^*\right)^T\right]^{-1} \tag{10}$$

$$P_{k|k} = P_{k|k-1} - K_k\left[H_k^* P_{k|k-1}\left(H_k^*\right)^T + R_k^* + H_k^* C_k + C_k^T\left(H_k^*\right)^T\right] K_k^T \tag{11}$$

where

$$\begin{aligned} R_k^* &= E\left[\nu_k^*\left(\nu_k^*\right)^T\right] \\ &= E\left[\left(\psi_k B_{k-1} A_k^{-1} \omega_{k-1} + \varsigma_k\right)\left(\psi_k B_{k-1} A_k^{-1} \omega_{k-1} + \varsigma_k\right)^T\right] \\ &= \psi_k B_{k-1} A_k^{-1} Q_{k-1}\left(A_k^{-1}\right)^T B_{k-1}^T \psi_k^T + R_k \end{aligned} \tag{12}$$

Noting that H_k^*, ν_k^* contains the nonlinear approximation residuals in matrices B_k and A_k, in order to simplify the analysis we assume that $\beta_k = I$, which is valid for both loosely and tightly coupled GNSS/INS [15]. The state error is utilized as the output of estimator and a closed feedback KF loop is used to compensate the drift of inertial sensors, so $\alpha_k = I$ is also acceptable. Notice that, CKF with colored measurement noise has the same prediction stage