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Optimistic value based optimal control for uncertain linear singular systems and application to a dynamic input-output model

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ABSTRACT

In this paper, optimal control problems for uncertain discrete-time singular systems and uncertain continuous-time singular systems are considered under optimistic value criterion. The above singular systems are assumed to be regular and impulse-free, and optimistic value method is employed to optimize uncertain objective functions. Firstly, based on Bellman's principle of optimality, a recurrence equation is presented for settling optimal control problems subject to uncertain discrete-time singular systems. Then, by applying the principle of optimality and uncertainty theory, an equation of optimality for an optimal control model subject to an uncertain continuous-time singular system is derived. The optimal control problem can be settled through solving the equation of optimality. Two numerical examples and a dynamic input-output model are given to show the effectiveness of the results obtained.

1. Introduction

The study of optimal control greatly attracted the attention of many mathematicians for the necessity of strict expression form in optimal control theory. In past several decades, optimal control theory has achieved plenty of developments not only in theory but also in applications such as economics, production engineering and management. An optimal control problem for a given system is to choose the best decision such that an objective function is optimized. Kirk [1] investigated optimal control theory and obtained lots of meaningful results. Bryson, Ho and Siouris [2] studied optimal control problems in various aspects including optimization, estimation and control. Naidu [3] considered singular perturbations and time scales in control theory and its applications. Then Das and Mahanta [4] proposed a chattering free optimal second order sliding mode control to stabilize nonlinear systems.

From 1970s lots of researchers began to investigate stochastic optimal control problems, such as in Merton [5] for finance. In recent decades, the study of stochastic optimal control has been made considerable developments, for example, Fleming and Rishel [6], Harrison [7], Karatzas [8] and Cairns [9] studied optimal control problems of Brownian motion or stochastic differential equations and applications in finance and engineering. One of the main methods to handle optimal control problems is Bellman's dynamic programming. The utilization of dynamic programming in optimization over Ito's process was discussed in Dixit and Pindyck [10].

The complexity of the real world makes the events we face uncertain in various forms. Plenty of human uncertainty does not behave like randomness, such as the price of a new stock, oil filed reserves and bridge strength. In order to deal with these phenomena, an uncertainty theory was established in [11] and refined by Liu [12] as a branch of axiomatic mathematics for modeling human uncertainty. Furthermore, Liu [13] introduced uncertain process and canonical process as counterparts of stochastic process and Wiener process, respectively. Then the concept of uncertain differential equation was presented in [13].

Based on uncertain differential equation, Zhu [16] studied the excepted value model of uncertain optimal control problem. Employing Bellman's principle of optimality, an equation of optimality was derived as a counterpart of HJB equation, and then he solved an uncertain portfolio selection problem. Moreover, by the equation of optimality, Yao and Qin [17] proposed an uncertain linear quadratic control model. Then Xu and Zhu [18], Kang and Zhu [19] studied uncertain bang-bang control problems for continuous-time system and multi-stage system, respectively. Gao and Yu [20] employed expected value criterion and optimistic value criterion to investigate a finite extensive game. Sheng and Zhu [21] discussed the optimistic value model of uncertain optimal control.

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As we all know, expected value is the weighted average of uncertain variables in the sense of uncertain measure. However, sometimes we need to take other characters of uncertain variables into account. For example, if individual incomes in a city appear two-level differentiation phenomenon, and the difference between higher income and lower income is too large, in this case average income may not be discussed alone. Then optimistic value of individual incomes may be considered. We may study the problem such as that 90% of the individual incomes achieve how many dollars above.

Singular systems, also known as descriptor systems, implicit systems and generalized state-space systems, are described by differential-algebraic equations (DAEs). Singular systems [22,23] have been extensively studied during the past decades due to the fact that they are able to describe plenty of natural phenomena in physical systems such as economics, demography, microelectronic circuits and so on [24–26]. Liu, Lin and Chen [27] studied the admissibility problem for a class of linear singular systems with time-varying delays. However, because of the difficulty appearing in analysis, few results are concerned with the optimal control of singular systems.

One of the contributions of this paper is to transform an uncertain linear singular system to two uncertain sub-systems which are simpler for investigation. This result inspired by the work in [30] is presented in Section 3. Getting a recurrence equation for an optimal control problem subject to an uncertain discrete-time singular system is the second contribution. Compared with the work in [19], this recurrence equation is able to settle more discrete-time optimal control problems and has more applications in practice. The last one is an equation of optimality derived in Section 5 which turns an optimal control problem for an uncertain continuous-time singular system to become an easier question. This work based on the results in [16,21] enlarges the scope of uncertain optimal control.

Inspired by the previous works, we will investigate optimal control problems for uncertain discrete-time singular systems and uncertain continuous-time singular systems under optimistic value criterion. The organization of this paper is as follows: in the next section, we will review some concepts such as uncertain measure, uncertainty space, optimistic value of uncertain variable, uncertain process, canonical process, and uncertain differential equation. In Section 3, we will introduce a kind of equivalent restrict form of an uncertain discrete-time singular system provided that it is regular and impulse-free. Based on this equivalent restrict form and Bellman's principle of optimality, a recurrence equation will be presented for settling an optimal control problem subject to an uncertain discrete-time singular system. In Section 4, we will obtain bang-bang optimal controls for two different types of uncertain discrete-time singular systems. In Section 5, we will consider an optimal control problem for an uncertain continuous-time singular system and get an equation of optimality for solving such problem. Two numerical examples and a dynamic input-output model are provided to illustrate the results obtained in Sections 4 and 5.

2. Preliminary

In convenience, we give some useful concepts at first. Let Γ be a nonempty set, and $\mathcal{L}$ a σ-algebra over Γ. Each element $\Lambda \in \mathcal{L}$ is called an event.

Definition 2.1 ([11]). A set function $\mathcal{M}$ defined on the σ-algebra $\mathcal{L}$ is called an uncertain measure if it satisfies the following four axioms:

Axiom 1. **(Normality)** $\mathcal{M}\{\Gamma\} = 1$;
Axiom 2. **(Self-Duality)** $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$ for any event Λ;
Axiom 3. **(Countable Subadditivity)** $\mathcal{M}\left\{\bigcup_{i=1}^{\infty}\Lambda_i\right\} \le \sum_{i=1}^{\infty}\mathcal{M}\{\Lambda_i\}$ for any events $\Lambda_1, \Lambda_2, \cdots$.

Definition 2.2 ([11]). Let Γ be a nonempty set, $\mathcal{L}$ the σ-algebra over Γ, and $\mathcal{M}$ an uncertain measure. Then the triplet $(\Gamma, \mathcal{L}, \mathcal{M})$ is said to be an uncertainty space. An uncertain variable is a measurable function ξ from an uncertainty space $(\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for any Borel set of real numbers, the set

$$\{\xi \in B\} = \{\gamma \in \Gamma | \xi(\gamma) \in B\}$$

is an event.

Definition 2.3 ([11]). The uncertainty distribution $\Phi: \Re \to [0, 1]$ of an uncertain variable ξ is defined by

$$\Phi(x) = \mathcal{M}\{\xi \le x\}.$$

Definition 2.4 ([14]). The uncertain variables $\xi_1, \xi_2, \ldots, \xi_m$ are said to be independent if

$$\mathcal{M}\left\{\bigcap_{i=1}^{m}\{\xi_i \in B_i\}\right\} = \min_{1\le i\le m}\mathcal{M}\{\xi_i \in B_i\}$$

for any Borel sets $B_1, B_2, \ldots, B_m$ of real numbers.

Independence of uncertain variables does not mean they are unrelated but that no "new" information about any uncertain variables can be obtained through observations of the others. For instance, uncertain variables are independent if they are defined on different uncertainty spaces.

Definition 2.5 ([11]). Let ξ be an uncertain variable. Then the expected value of ξ is defined by

$$E[\xi] = \int_0^{+\infty}\mathcal{M}\{\xi \ge r\}dr - \int_{-\infty}^{0}\mathcal{M}\{\xi \le r\}dr$$

provided that at least one of the two integrals is finite.

Definition 2.6 Let ξ be an uncertain variable with finite expected value e. Then the variance of ξ is defined by $V[\xi] = \mathcal{E}[(\xi - e)^2]$.

Definition 2.7 ([11]). Let ξ be an uncertain variable, and $\alpha \in (0, 1]$. Then $\xi_{\sup}(\alpha) = \sup\{r | \mathcal{M}\{\xi \ge r\} \ge \alpha\}$ is called the α-optimistic value to ξ; and $\xi_{\inf}(\alpha) = \inf\{r | \mathcal{M}\{\xi \le r\} \ge \alpha\}$ is called the α-pessimistic value to ξ.

"sup A" means the supremum of a given set A, and "inf" the infimum of set A.

Theorem 2.1 ([11,12]). *Assume that ξ is an uncertain variable. Then we have*

(a) *if $\lambda \ge 0$, then $(\lambda\xi)_{\sup}(\alpha) = \lambda\xi_{\sup}(\alpha)$, and $(\lambda\xi)_{\inf}(\alpha) = \lambda\xi_{\inf}(\alpha)$;.*
(b) *if $\lambda < 0$, then $(\lambda\xi)_{\sup}(\alpha) = \lambda\xi_{\inf}(\alpha)$, and $(\lambda\xi)_{\inf}(\alpha) = \lambda\xi_{\sup}(\alpha)$.*
(c) *$(\xi + \eta)_{\sup}(\alpha) = \xi_{\sup}(\alpha) + \eta_{\sup}(\alpha)$ if ξ and η are independent.*

Based on the uncertainty space, Liu introduced the concepts of uncertain process, canonical process, uncertain differential equation, and etc.

Definition 2.8 ([14]). Let T be an index set and let $(\Gamma, \mathcal{L}, \mathcal{M})$ be an uncertainty space. An uncertain process is a measurable function from $T \times (\Gamma, \mathcal{L}, \mathcal{M})$ to the set of real numbers, i.e., for each $t \in T$

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