ARTICLE IN PRESS

ISA Transactions ■ (■■■■) ■■■–■■■

Contents lists available at ScienceDirect

ISA Transactions

journal homepage: www.elsevier.com/locate/isatrans

Research article

A novel linear tracking integrator with integral compensation and its application

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A R T I C L E   I N F O

Article history:
Received 9 December 2016
Received in revised form
19 August 2017
Accepted 13 September 2017

Keywords:
Linear tracking integrator
Finite-time convergent integral observer
Integral action
Noisy measurement
Describing function
Frequency-domain

A B S T R A C T

A novel linear tracking integrator (LTI) with integral compensation is proposed for efficient integral estimation from a contaminated measurement with a constant or time-varying bias. The limitation of finite-time convergent integral observer (FTCIO) in ruling out the integral drift is firstly revealed via describing function method. Subsequently, by the utilization of integral action in the feedback path, a simple but effective linear tracking integrator is established to provide a practical solution in achieving a drift-free integral estimate. The highlight is that the proposed LTI can simultaneously give the accurate integral and tracking estimates from a noisy measurement without relying on the condition of observability. In addition, frequency-domain analysis of LTI is investigated to give a viable guideline of parameter tuning. Illustrative simulations and comparison with Kalman filter are included to demonstrate the superiority of LTI in accomplishing precise integral tracking in the presence of constant or time-varying bias. Finally, the effectiveness of LTI is also confirmed by an application on autopilot design for aircraft.

1. Introduction

Integral operation, denoted as $1/s$ operator, has ever played a vital role in almost all engineering disciplines and the necessity to obtain the time integral from a measurement has exerted tremendous fascinations from diverse sectors for engineering applications, including high precision motion control [1–4,28–30], inertial navigation system (INS) [5–8], among others. To be specific, in motion control applications, to realize the velocity tracking, the velocity state is usually obtained via integrating the available accelerometer signal directly. However, this approach fails in practice because of the imperfection associated with accelerometer device such as the dc offset, resulting in an unbounded error in the integral. Thus, some advanced state observers, such as kinematic Kalman filter [2], two-channel approach [3], as well as a dynamically compensated velocity observer [4], are constructed by effectively integrating the position signal given by a high-resolution encoder with an acceleration measurement, based on which, a noise-tolerant and drift-free velocity information can be available. Regarding the inertial navigation system, both velocity and position of the rigid body are typically calculated by integrating the accelerometer output once and twice in the navigation frame. Similarly, the orientation with respect to a known starting point is usually determined by the integral of the sensed angular velocity [5–7]. However, for a long-time navigation, the integral drift caused by the unknown bias in the accelerometer measurement and inappropriate integration will lead to the failure of prescribed navigation task. Therefore, GPS module that gives the position-related information, is usually incorporated with INS to form a complementary navigation system to counteract the drift phenomena. For the aforementioned scenarios, the implementation of efficient velocity estimation schemes is only confined to the case where the observed dynamic structure of process satisfies the condition of observability, and requires not only imperfect accelerometers, but also additional position sensors. However, in many applications, due to implementation cost reduction, structure restriction and weight limitation, the required position information would be otherwise unavailable or severely disturbed by the noisy environment. Furthermore, heavy frequency noise in position sensors may also deteriorate the estimation quality of velocity signal. Consequently, how to obtain the velocity estimate from the acceleration signal alone, while achieving good performance in eliminating the integral drift is an open and challenging issue.

Real-time and physically realizable methods for integration can be divided into two categories: analog integrators [9,10] and digital

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http://dx.doi.org/10.1016/j.isatra.2017.09.011
0019-0578/

Please cite this article as: Shao X, et al. A novel linear tracking integrator with integral compensation and its application. ISA Transactions (2017), http://dx.doi.org/10.1016/j.isatra.2017.09.011

integrators [11–16]. For analog integrators, which place a capacitor in feedback across an op-amp, cannot integrate beyond the supply rail voltage due to saturation [9]. In addition, they are sensitive to the temperature of operating environment and parameter variations in resistances or capacitors. Besides, analog integrators tend to drift, exhibit low precision in response to unbalanced input bias currents [10]. On the other hand, digital integrators are performed numerically via quadrature that involves sums. The problem of numerical integration is of estimating the value $I=\int_a^b f(t)dt$ with $[a, b]$ being a finite closed interval. The trapezoidal and the Simpson rules are among the most popular methods for approximating the evaluation of the definite integrals [11]. For example, a novel wideband third-order trapezoidal integrator is derived by applying the z-transform technique to the closed-form Newton-Cotes integration formula and has the feature of accurately approximating the ideal integrator over the whole Nyquist frequency range [12]. Moreover, the well-known class of digital integrators can also be derived from a classical continuous-time approximate differentiator by using bilinear transformation [13]. It is apparent that the existing literatures [11–16] on digital integrators are mainly focused on applying different numerical integration rules to approximate integral polynomials. However, they are essentially approximations of the definite integral [14], thus limited in restraining the accumulation of drift effect in the integral.

Different from the above design philosophy of digital integrators, some interesting and pioneering contributions dedicated to integral estimation are recently introduced in [8,17,18]. A high-order nonlinear continuous integral-derivative observer is reported in [17] by means of finite-time stability to obtain the multiple integrals and the derivatives of a signal with almost no drift phenomena and sufficient stochastic noise rejection ability. As a special case of [17], the finite-time convergent double-integral observers are constructed and have been successfully extended to the challenging application of quadrotor control [18] and navigation [8]. However, for the existing results, although the stability is rigorously established in time-domain via finite-time stability, the principle of the integral observer in rejecting the output disturbance is not straightforwardly explored using frequency-domain analysis. Furthermore, the steady-state response of integral error with respect to different magnitudes of unknown bias is much more important in practice and deserves further investigation.

Inspired by the above observations, to overcome the aforementioned weakness associated with the finite-time convergent integral observer (FTCIO), a novel linear tracking integrator (LTI) with integral compensation is proposed for efficient integral estimation from a contaminated measurement with a constant or time-varying bias. The main contributions in this paper are summarized as follows.

- The limitation of FTCIO against bias in the measurement is firstly revealed via describing function method. To overcome the disadvantage of FTCIO, and inspired by the fact that the integral control is effective in removing the offset between the actual and desired signal, together with a linearized variant of FTCIO, a simple but effective linear tracking integrator is established to effectively eliminate the drift effect, which can be treated as an extension of theoretical results on FTCIO.
- The steady-state response of integral error with respect to different magnitudes of unknown bias is investigated with the help of the final-value theorem. In addition, frequency domain analysis of LTI is investigated to give a viable guideline of parameter tuning.
- The existing state observers, such as Kalman filter and extended state observer(ESO), cannot be applied to handle the integral estimation due to the lack of observability. However, the proposed LTI is constructed under a different methodology, and its unique characteristic in obtaining the integral estimate from acceleration signal sets LTI apart from the available observers.
- Illustrative simulations and comparison with Kalman filter are included to demonstrate the effectiveness of LTI in achieving a satisfactory integral performance. And an application on autopilot design for aircraft is also given.

The paper is organized as follows. The recalling of finite-time convergent integral observer is illustrated in Section 2. The main results on the proposed linear tracking integrator is presented in Section 3. In Section 4, frequency domain analysis of the proposal is given. Illustrative simulations and comparison with Kalman filter are shown in Section 5. Application on autopilot design is provided in Section 6.The paper ends with a few concluding remarks in Section 7.

2. The recalling of finite-time convergent integral observer

Consider the following second-order cascaded integrator system with a single output y,

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\underbrace{\begin{bmatrix}0&1\\0&0\end{bmatrix}}_{\mathbf{A}}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\underbrace{\begin{bmatrix}0\\1\end{bmatrix}}_{\mathbf{B}}\dot{a}(t)$$
$$y(t)=\underbrace{\begin{bmatrix}0&1\end{bmatrix}}_{\mathbf{C}}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+n(t)+d(t) \tag{1}$$

where y denotes the sensed acceleration signal with n, d being zero-mean Gaussian stochastic noise and unknown bias, respectively. And $x_1, x_2, \dot{a}$ represent the real velocity, acceleration signal and its time derivative, respectively. Our design objective is concerned with the question of, whether or not the accurate velocity state without integral drift can be generated from the sensed acceleration signal alone, the positive answer would have significant implications on the synthesis of control schemes with high-quality velocity information required. For the existing digital integrators, they are limited in calculating the drift-free integral estimate. In addition, although the advanced state observers or differentiators investigated in the literature [19–21] are capable of extracting the derivative states or disturbances acted upon the system accurately from the input-output data, it is still insufficient to apply the existing observers to tackle the integral estimation task.

Remark 1. To achieve the goal of extracting the integral estimate from the acceleration signal, it is necessary to check whether or not the observed dynamics satisfies the observability condition, the property of which is the premise of constructing a state observer. The existing observers, including Kalman filter and ESO, can be implemented and available only when the observed dynamics meets the above requirement. However, since the observability matrix of the considered system (1) with acceleration measurement alone is not of full rank, thus the Kalman filter and ESO cannot guarantee the convergence of velocity estimate even in the absence of bias.

Recently, without relying on the condition of observability, a high-order nonlinear continuous integral-derivative observer is proposed in [17] by means of finite-time stability to obtain the multiple integrals. For the 1-fold integral estimation problem, a particular form of nonlinear continuous integral-derivative observer, termed as the finite-time convergent integral observer (FTCIO), can be readily derived from [17], and the following corollary gives the specific form of FTCIO:

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