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Pole-placement Predictive Functional Control for under-damped systems with real numbers algebra

K. Zabet [a], J.A. Rossiter [b], R. Haber [a,*], M. Abdullah [b]

[a] Cologne Univ. of Applied Sciences, Inst. of Plant and Process Engineering, Betzdorfer Str. 2, D-50679 Koeln, Germany
[b] Dept. of Automatic Control and Systems Eng., University of Sheffield, S1 3JD, UK

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ABSTRACT

This paper presents the new algorithm of PP-PFC (Pole-placement Predictive Functional Control) for stable, linear under-damped higher-order processes. It is shown that while conventional PFC aims to get first-order exponential behavior, this is not always straightforward with significant under-damped modes and hence a pole-placement PFC algorithm is proposed which can be tuned more precisely to achieve the desired dynamics, but exploits complex number algebra and linear combinations in order to deliver guarantees of stability and performance. Nevertheless, practical implementation is easier by avoiding complex number algebra and hence a modified formulation of the PP-PFC algorithm is also presented which utilises just real numbers while retaining the key attributes of simple algebra, coding and tuning. The potential advantages are demonstrated with numerical examples and real-time control of a laboratory plant.

1. Introduction

PFC (Predictive Functional Control) [1] is probably the most successful industrial implementation of MPC (Model Predictive Control) based on the numbers and breadth of applications. The main reason for this is relatively simple in that the coding requirements are similar to that for PID (Proportional-Integral-Derivative) controllers and thus the PFC strategy is a competitor with PID rather than more expensive plant wide or system wide approaches. Moreover, it has some advantages over PID in that the tuning mechanism is intuitive being based mainly on a desired time constant (equivalently settling time or convergence rate) and also it embeds a reasonable level of systematic constraint handling using relatively low computational complexity.

Nevertheless, the main weakness of conventional PFC is the same as its strength, that is the relative simplicity [2,3]. Although execution and coding are straightforward for systems with over-damped or simple dynamics, a different picture emerges with systems with less desirable open-loop dynamics [4]. Consequently, although a conventional PFC [1] can work with systems of integrators, open-loop unstable processes and non-minimum-phase characteristics, often the tuning is difficult and the implementation less simple and intuitive. Thus one purpose here is to develop a modified PFC approach which retains the core attributes of simplicity but more specifically, retains intuitive insight during the design which means the approach is simple for technicians to deploy.

Predictive control algorithm can be calculated by properly planning the manipulated signal sequence via minimizing a cost function. The idea of pole-placement design for predictive control is not new. Pole-placement state-feedback design for optimizing continuous-time predictive control was applied in [5] and extended this algorithm for the constrained case in [6]. GPC (Generalized Predictive control) [7] has two degrees of freedom and allows a design based on pole-placement, see [8] and [9]. Investigations of the stability of PFC for first-order process models [10] were followed by a pole-placement PFC controller recommended for higher-order, over-damped processes in [11].

This paper has a focus on systems with significant under-damped dynamics in the open-loop and first considers the efficacy of a routine PFC implementation. It is demonstrated via a number of examples, that the efficacy is variable which motivates the need for an improved algorithm. Earlier literature has discussed the possibility of shaping the input predictions [4], but although often effective, that approach has the disadvantage of requiring some moderately difficult algebra/coding and there is still a need to fully understand the robustness to uncertainty of such approaches. This paper takes an alternative approach which is to explore and develop a recently proposed alternative the PP-PFC (Pole-Placement PFC) [11]. The main contribution here is to consider the extent to which this approach is suitable for handling under-damped

* Corresponding author.
E-mail addresses: j.a.rossiter@sheffield.ac (J.A. Rossiter), robert.haber@th-koeln.de (R. Haber).

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systems. Moreover, as will be seen, a secondary benefit is additional flexibility in the choice of target poles to include mild under-damping; such an option is not available to conventional PFC.

A simplistic implementation of the proposed PP-PFC algorithm for underdamped systems is shown to rely on complex number algebra and this has some possible negative consequences. Firstly, the computational effort is slightly greater, although that could be considered trivial in practice. Secondly however, the requirement for complex number algebra in itself could be a problem as many low level process control units (where PFC would be applied alongside competitor approaches such as PID) do not support complex number algebra. In view of these observations, a second contribution of this paper is to propose algorithms which circumvent the complex number algebra in a relatively simple fashion, thus allowing straightforward coding, maintenance and tuning.

Section 2 will give a basic background on conventional PFC and demonstrate the potential difficulties when applying this to under-damped systems. Section 3 will introduce the pole-placement PFC approach for systems with real poles followed by Section 4 which will discuss how this approach is extended to cope with complex poles, that is under-damped systems. Section 5 will then develop an alternative formulation of PP-PFC which uses just real number algebra. Section 6 gives numerical examples and also some simulations on hardware.

2. Background of PFC

This section gives a brief review of a basic PFC algorithm and demonstrates a normal tuning procedure.

2.1. PFC concepts

The basic principle underlying PFC approaches is that the desired output dynamic is close to that of a first-order response with a specified pole λ. The hope is that if one, recursively at each sample, ensures the prediction of the system behavior is close to the desired dynamic, then the closed-loop behavior is likely to be close to that dynamic. Hence, for a desired steady-state set value of r, a typical target trajectory r^*, expressed in discrete time, takes the form[1]:

$$r^*(k) = \frac{(1-\lambda)z^{-1}}{1-\lambda z^{-1}} r(k). \tag{1}$$

In the interest of simple computation, PFC differs from more standard MPC approaches in that it uses the prediction at just a single point, the so called coincidence horizon, here denoted by a n_y step ahead prediction. The control law is defined by forcing the system prediction to match the target dynamic of $r^*(k)$ at a point n_y steps ahead, as illustrated in Fig. 1.

In practice, the system output $y_p(k)$ is not beginning from zero, so the target trajectory is one which follows a first-order dynamic from the current point $y_p(k)$ to the correct steady-state, that is:

$$r^*(k+i) = r(k) - \lambda^i[r(k) - y_p(k)], \quad i \geq 1. \tag{2}$$

PFC is defined by forcing coincidence n_y steps ahead and thus the control law is defined from the equality:

$$y_p(k+n_y) = r(k) - \lambda^{n_y}[r(k) - y_p(k)]. \tag{3}$$

[1] In the following the case of a stepwise change in the reference signal is assumed. The same algorithm works for stepwise change in the output additive disturbance, as well.

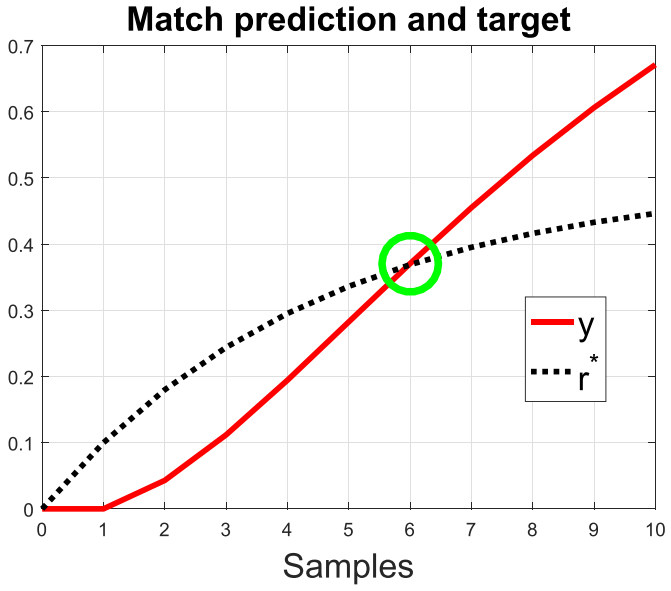

Fig. 1. Illustration of PFC target dynamic r^* and coincidence of the output prediction y_p with target dynamic $n_y = 6$ samples ahead.

Mismatch between process output y_p and model output y_m is assumed constant during the prediction horizon and hence offset-free tracking can be achieved with a minor modification to take account of this bias. The system prediction is given by the model prediction plus an estimated disturbance $d(k)$ (variants of this exist but are not central to the current paper):

$$y_p(k+n_y) = y_m(k+n_y) + d(k), \quad d(k) = y_p(k) - y_m(k). \tag{4}$$

Simplification 1. The n_y steps ahead prediction $y_p(k+n_y)$ depends upon the future choices of control actions. As PFC is premised on being as simple as possible, a typical assumption is that the future inputs remain constant, that is $(u(k+i) = u(k), \quad i \geq 1)$. This has the advantage that only one decision variable is needed so the desired selection to satisfy (3) is straightforward to code (this also applicable with non-linear processes).

Simplification 2. In order to maintain simple coding, PFC overcomes the complexity of prediction algebra by using partial fractions to express the nth-order model $G_m(z)$ as a sum of first-order models [1,2,12] and hence:

$$\left.\begin{aligned} y_m(k) &= G_m(z)u(k), \\ G_m(z) &= \sum_{i=1}^{n} G_i(z), \end{aligned}\right\} \Rightarrow y_m(k) = \sum_{i=1}^{n} G_i(z)u(k) = \sum_{i=1}^{n} \frac{b_i z^{-1}}{1+a_i z^{-1}} u(k). \tag{5}$$

The effective structure of the model is illustrated in Fig. 2 where G_p represents the real (unknown) process and G_i denote the partial fraction expansion of the assumed model $G_m(z)$. In practice this means that the independent model deployed in PFC code comprises a number of first-order independent models running in parallel; clearly the coding and computation requirement for each is trivial.

The advantage of this parallel formation is that n_y steps ahead predictions can be defined explicitly and without the need for costly or cumbersome prediction algebra [13]. To be precise, the predictions for the model can be expressed as the sum of the predictions of a number of first-order models with component outputs $y_m^{(i)}$, that is:

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