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Research article

Coordinated tracking of linear multiagent systems with input saturation and stochastic disturbances ☆

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ABSTRACT

This paper addresses the coordinated tracking problem for linear multiagent systems with input saturation and stochastic disturbances. The objective is to construct a class of tracking control laws that achieve consensus tracking in the absence of disturbances, while guaranteeing a bounded variance of the state difference between the follower agent and the leader in the present of disturbances, under the assumptions that each agent is asymptotically null controllable with bounded controls (ANCBC) and the network is connected. By using the low gain feedback technique, a class of tracking control algorithms are proposed, and the coordinated tracking problem is solved through some routine manipulations. Finally, numerical examples are provided to demonstrate the theoretical results.

1. Introduction

During the last several decades, coordinated control of multiagent systems have received considerable attention due to its wide applications, see, for instance, [6,3,18,15,2,43,46,9,13,10,47], and the survey papers [27,23] and references therein for more details and developments. Many researchers have done much effort on coordinated tracking of multiagent systems, and various coordinated tracking algorithms have been developed, including the consensus tracking algorithm and the swarm tracking algorithm. The consensus tracking means a group of agents, only relying on their neighbors' information, precisely reach the motion of the leader, while the swarm tracking means a group of agents, only relying on their neighbors' information, reach in a predesigned set of the motion of the leader. Recent efforts of coordinated tracking are mainly focused on multiagent systems with linear and nonlinear systems [21,19,44,12,42,25,40] while previous efforts were mainly on first- and second-order dynamics [7,41,34,5].

Input saturation has been widely considered owing to its universal existence in control systems. There have been plenty methods proposed for the stabilization problem of a single linear system with input saturation [35,36,16,48]. Among them, the low gain feedback [17,14] is an effective method to design a class of linear control laws in the semi-global stabilization framework. The feature of the low gain feedback is that given any bounded set of initial conditions, the closed-loop system can always stay in the linear domain by scheduling the feedback gain parameter small enough. Recently, the low gain feedback has been extended to treat the coordinated tracking problem of multiagent systems. By using low gain feedback, the semi-global consensus tracking for linear multiagent systems with actuator saturation has been investigated in [32,38,39]. Meanwhile, there are other works focused on coordinated tracking problem with saturation constraints from different perspectives. For example, in multi-agent systems, the coordinated tracking problem with input saturation has been studied for single-integrators [11] and double-integrators [26,45,1], respectively. In [20], the global consensus tracking problem was solved for high-order multiagent systems with input saturation. By using low and high gain feedback method, the semi-global coordinated tracking of linear multi-agent systems with input saturation was studied in [31]. With a modified algebraic Riccati equation, the problem of semi-global leader-following synchronization of discrete-time linear agents with actuator saturation is considered in [4]. It is worth to mention that disturbance uncertainties also universal exist in control systems. Generally, disturbance uncertainties can be divided into two types,

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that is, input-additive disturbances and non-input-additive disturbances. For the input-additive disturbances, the disturbances can be shown in form of $\delta(u+\omega)$, while the form of $\delta(u)+\omega$ represents the non-input-additive disturbances, where $\delta(\cdot)$ is the saturation function, u is the control input, and ω is the disturbances. It is known [28] that the general ANCBC systems with input saturation can achieve stability with a linear control law when the disturbance is input-additive, while in general [30], it is impossible to achieve stability via a linear control law in the presence of the non-input-additive disturbance. This implies that the picture is complicated when considering the non-input-additive disturbance. As for the multiagent systems, some previous works have considered both input saturation and input-additive disturbances. For example, the global coordinated tracking for linear multiagent systems with input saturation and input-additive disturbances was shown to be achievable in [33,37]. Although much effort has been made to solve the coordinated tracking problem for linear multiagent systems with both input saturation and input-additive disturbances, to the best of our knowledge, stochastic disturbances, the non-input-additive disturbances, has not been considered in the existent results.

With the above observations, in this paper, we deal with the coordinated tracking for a group of agents with input saturation and stochastic disturbances. By using the scheduled low gain design technique, we propose a class of tracking control algorithms that can always achieve consensus tracking in the absence of disturbances and guarantee a bounded variance of the state difference between the follower agent and the leader in the present of disturbances, under the assumptions that each agent is ANCBC and the network is connected. The major contributions of this paper are twofold. Firstly, compared with the results in [33,37] with input-additive disturbances, the disturbances discussed in this paper are non-input-additive, which implies that the scenario of the stability analysis of multiagent systems is more complex. Secondly, to the best of our knowledge, this is the first attempt to take the stochastic disturbances into account for coordinated tracking control of multiagent systems. Consequently, one of the main challenges is how to properly deal with stochastic and non-input-additive disturbances for achieving bounded variance in the controlled multiagent system. It is appropriate to emphasize that it is not clear that such coordinated tracking algorithms will exist, and we provide an explicit coordinated tracking design method based on the low gain feedback and a scheduling parameter.

The rest of the paper is organized as follows. Section 2 formulates the coordinated tracking problem. Section 3 presents the main results on the coordinated tracking. Section 4 provides simulation examples for verifying the theoretical results. Finally, some conclusion remarks are given in Section 5.

Before closing this section, some notations will be stated here. The notation $\text{diag}\{\omega_1, \omega_2, \dots, \omega_n\}$ denotes a diagonal matrix whose diagonal entries are ω_i, $i = 1, 2, \dots, n$. $\text{col}\{x_1, x_2, \dots, x_n\}$ and 1_N denote a column vector and an N-dimensional column vector with all elements of 1, respectively. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. The notation $P > 0$ (≥ 0) means that P is a real symmetric positive (semi-positive) definite matrix. $I_n \in \mathbb{R}^{n\times n}$ and 0 represent, respectively, the identity matrix and zero matrix. The set of real numbers is denoted by $\mathbb{R}$. The set of real-valued vectors of length m is given by $\mathbb{R}^m$. The set of real-valued $m \times n$ matrices is given by $\mathbb{R}^{m\times n}$.

2. Problem formulation

In the beginning, the graph theory will be introduced. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ be a weighted directed graph, where $\mathcal{V} = \{v_1, v_2, \dots, v_N\}$ is a finite nonempty set of nodes and edges $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ is the weighted adjacency matrix, where a_{ij} is the coupling strength of the directed edge (j, i) satisfying $a_{ij} > 0$ if (j, i) is an edge of $\mathcal{G}$ and $a_{ij} = 0$ otherwise. Let $N_i = \{j \in \mathcal{V}: (j, i) \in \mathcal{E}\}$ be the set of neighbors of node i in $\mathcal{G}$. Let $d_i = \sum_{j=1}^{N} a_{ij}$ the in-degree of vertex i, and $D = \text{diag}\{d_1, \dots, d_N\}$ the in-degree matrix of $\mathcal{G}$. The Laplacian matrix $L = [l_{ij}]$ of weighted digraph $\mathcal{G}$ is defined by $L = D - \mathcal{A}$. For any pair of vertices (i, j), if $a_{ij} = a_{ji}$, the graph is called an undirected graph. In this case, $\mathcal{G}$ is connected if there is a directed path between any pair of distinct nodes; otherwise, it is termed a directed graph. A directed tree is a directed graph, where every node, except the root, has exactly one parent. A spanning tree of a directed graph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph has (or contains) a spanning tree if a subset of the edges forms a spanning tree.

Given a graph $\mathcal{G}$, endow each of its N agents with a state vector $x_i \in \mathbb{R}^n$ and a control input $u_i \in \mathbb{R}^m$, and we consider each agent, labeled from 1 to N, is described by a stochastic differential equation

$$dx_i(t) = Ax_i(t)dt + B\delta(u_i(t))dt + Ed\omega_i(t), \quad i = 1, 2, \dots, N, \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $E \in \mathbb{R}^{n\times l}$, the disturbance $\omega_i \in \mathbb{R}^l$ is a Wiener process (a Brownian motion) with mean 0 and rate Q_i, the initial condition $x_i(0)$ is a Gaussian random vector which is independent of ω_i, and $\delta(\cdot)$ is a standard saturation function given as

$$\delta(u) = \begin{cases} u, & \text{if } |u| \leq 1 \\ \text{sign}(u), & \text{if} |u| > 1 \end{cases}.$$

The dynamics of the leader agent, indexed by 0, is described by

$$dx_0(t) = Ax_0(t)dt. \tag{2}$$

Let $\overline{\mathcal{G}}$ be a graph generated by graph $\mathcal{G}$ and the leader agent. If agent i is a neighbor of the leader at time t, then denote it by $b_{i0} > 0$, otherwise $b_{i0} = 0$. We also assume that $b_{i0} > 0$ for at least one i, for all $i = 1, 2, \dots, N$. Let L be the Laplacian of an undirected graph $\mathcal{G}$ consisting of N agents, and denote $B_0 = \text{diag}\{b_{10}, b_{20}, \dots, b_{N0}\}$, $b_0 = \text{col}\{b_{10}, b_{20}, \dots, b_{N0}\}$ and $0 < \lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_N$ the N eigenvalues of matrix $L + B_0$, which can be easily obtained according to [7].

It is known from [35] that a linear system with input saturation can be stabilized if and only if (A, B) is stabilizable, and all eigenvalues of A are in the closed left-half plane. Without loss of generality, we assume that the pair (A, B) is given in the following form:

$$A = \begin{bmatrix} A^0 & 0 \\ 0 & A^- \end{bmatrix}, \quad B = \begin{bmatrix} B^0 \\ B^- \end{bmatrix}$$

where A^- contains all eigenvalues of A that have negative real parts, and A^0 contains all eigenvalues of A that are on the imaginary axis. The stabilizability of (A, B) then implies that (A^0, B^0) is controllable. Clearly, (A^-, B^-) does not affect the stabilizability property of the system. In what follows, without loss of generality, we will further give an assumption:

Assumption 1. The pair (A, B) is controllable and all the eigenvalues of A are on the imaginary axis.

Remark 1. According to Assumption 1, no exponentially unstable mode exists in A. It is known that Assumption 1 is actually a necessary condition for stabilizing linear systems with input saturation by low gain feedback [16].

Assumption 2. The generated graph $\overline{\mathcal{G}}$ consisting of the follower agents and the leader contains a directed spanning tree rooted at the leader.

Remark 2. Assumption 2 is quite general for multiagent systems [37,38], and it is reasonable that for the leader-follower