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Consensus of a class of discrete-time nonlinear multi-agent systems in the presence of communication delays

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ABSTRACT

In this paper, we study the consensus problem for a class of discrete-time nonlinear multi-agent systems. The dynamics of each agent is input affine and the agents are connected through a connected undirected communication network. Distributed control laws are proposed and consensus analysis is conducted both in the absence and in the presence of communication delays. Both theoretical analysis and numerical simulation show that our control laws ensure state consensus of the multi-agent system.

1. Introduction

Coordinated control of multi-agent systems has been inspired by the collective motion of animals in the nature. The study of coordinated control can be traced back to 1986 when Craig Reynolds simulated flocking of birds with his program named “Boids.” Coordinated control of multi-agent systems has received increasing attention for its potential in applications involving multiple networked systems, such as clustering of small satellites, wireless sensor networks and synchronization of networked oscillators.

Consensus control is one of the most important problems in coordinated control, which is concerned with reaching a network-wide agreement on either states or outputs of agents while each agent can only access limited information. References [1] and [2] provide overviews of works related to consensus of multi-agent systems.

Consensus control problem has been intensively studied for multi-agent systems with linear dynamics and many of the existing works address the issue of communication delays. In [3], a consensus algorithm was proposed for double integrator agents, and communication delays were considered. Ref. [4] and [5] study the consensus control problem of higher order multi-agent systems. In [4], it was demonstrated that, in the presence of arbitrarily large communication delays, the proposed truncated predictor feedback based controllers drive the multi-agent system into consensus. In [5], a more comprehensive model was studied, and both internal uncertainties and external disturbances were considered. Leader-following consensus of multi-leader multi-agent systems was studied in [6], where some observer-based consensus protocols were constructed.

Recently, more and more studies have addressed consensus control problems of nonlinear multi-agent systems. Both references [7] and [8] are concerned with leader-following consensus of multi-agent systems with unknown nonlinearities. In [7], system nonlinearities are assumed to be partially unknown and reinforcement learning was employed to achieve approximate optimal consensus. In [8], agents are in the Brunovsky form with completely unknown nonlinearities, which are estimated through two neural networks, and bounded unknown disturbances. In [9], the output consensus problem was investigated for a class of nonlinear multi-agent systems which are input affine and input-output passive. Outputs are synchronized when communication delay is absent, and delayed-output synchronization is achieved when the system is subject to constant communication delays. Leader-following consensus for multi-agent systems with Lipschitz-type nonlinear dynamics was addressed in [10]. Distributed control laws were developed under both fixed and switching communication topologies. The scenario where the switching communication topologies do not always contain a directed spanning tree also was investigated.

In many cases, controllers operate in discrete-time settings. However, as observed in [11] and [12], applications of continuous-time controllers through direct discretization could be very

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restrictive due to the requirement for small sampling periods. Therefore, for the application purpose, cooperative control algorithms should be designed in discrete-time settings as well. Flocking of multiple vehicles with discrete-time nonholonomic dynamics was studied in [13], where communication delays and collision avoidance were taken into consideration as well, and potential functional based low gain control laws were proposed. It was shown that the discrete-time closed-loop multi-agent system tolerates arbitrarily large bounded communication delays, while the corresponding continuous-time multi-agent system, studied in [14], can also tolerate arbitrarily large bounded communication delays. The consensus control problem for agents with discrete-time linear dynamics and time-varying delays was studied in [15], under a switching communication network. Ref. [16] addresses second order consensus of discrete-time linear multi-agent systems with nonuniform but bounded delays. Consensus of a group of linear agents subject to transmission nonlinearity has been studied in [17].

In this paper, we study the consensus problem of a class of multi-agent systems with discrete-time nonlinear affine-in-control dynamics. We note that the stabilization problem for such discrete-time control-affine nonlinear systems has been studied in [18] and that the consensus problem for multi-agent systems with control-affine nonlinear dynamics was studied in the continuous-time setting in our earlier work [19].

We will develop a low gain feedback method to counteract the effect of communication delays. In particular, a parametric Lyapunov equation approach has been proposed in [20] as a solution to stabilization of linear discrete-time systems with input delays. In this study, we will develop low gain control laws based on parametric Lyapunov equation for consensus control of multi-agent systems in the presence of communication delays. We will demonstrate that, both in the absence and in the presence of communication delays, state consensus is ensured under the resulting discrete-time control laws.

The remainder of this paper is organized as follows. We give the problem definition and assumptions in Section 2. Sections 3 and 4 present the distributed control laws and the analysis of the closed-loop multi-agent system in the absence and in the presence of communication delays, respectively. In Section 5, the effectiveness of the proposed control laws is demonstrated by numerical simulation. Section 6 draws the conclusion to the paper.

2. Problem statement

We consider a discrete-time multi-agent system that consists of N agents, each described by,

$$x_i(k+1) = Ax_i(k) + g(x_i(k))u_i(k), \tag{1}$$

where $x_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^m$ are the state vector of agent i and the control input, respectively, $A \in \mathbb{R}^{n\times n}$ and $g(x_i) \in \mathbb{R}^{n\times m}$.

Definition 1. Global consensus of a multi-agent system is said to be achieved if the states of all agents are synchronized, i.e., for all $x_i(0) \in \mathbb{R}^n$, $i = 1, 2, \ldots, N$, $\lim_{k\to\infty} \| x_i(k) - x_j(k) \| = 0$, $i, j = 1, 2, \ldots, N$.

The information exchange among the agents is represented by a graph introduced in the following definitions.

Definition 2. An undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$ consists of a nonempty set of nodes $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$, and a set of edges $\mathcal{E} \subset \mathcal{V}\times\mathcal{V}$. An unordered pair $(v_i, v_j) \in \mathcal{E}$ if there exists a bidirectional link between node v_i and node v_j.

Definition 3. Consider an undirected graph $\mathcal{G}$, described by its adjacency matrix $A_{adj} = [a_{ij}] \in \mathbb{R}^{N\times N}$, with

$$a_{ij} = a_{ji} = \begin{cases} 1, & \text{if } (v_i, v_j) \in \mathcal{E}, \\ 0, & \text{otherwise.} \end{cases}$$

We define the graph Laplacian matrix $L = D - A_{adj}$, where $D = \mathrm{diag}\{d_1, d_2, \ldots, d_N\}$ and $d_i = \sum_{j=1}^N a_{ij}$, $i = 1, 2, \ldots, N$. For an undirected graph, $L = L^{\mathrm{T}}$.

Lemma 1 ([1]). *The Laplacian matrix L of an undirected graph is positive semi-definite.*

Lemma 2 ([21]). *An undirected graph is connected if and only if zero is a simple eigenvalue of the Laplacian matrix L with $\bar{1}_N = [1, 1, \ldots, 1]^{\mathrm{T}} \in \mathbb{R}^N$ being the eigenvector associated with it.*

Neighborhood consensus errors are defined as

$$e_i(k) = \sum_{j=1}^N a_{ij}(x_i(k) - x_j(k)), \quad i = 1, 2, \ldots, N, \tag{2}$$

where a_{ij} has been defined in Definition 3.

We introduce the following notations,

$$x = \left[x_1^{\mathrm{T}}, \ x_2^{\mathrm{T}}, \ \ldots, \ x_N^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{Nn}, \tag{3}$$

$$e = \left[e_1^{\mathrm{T}}, \ e_2^{\mathrm{T}}, \ \ldots, \ e_N^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{Nn}, \tag{4}$$

$$u = \left[u_1^{\mathrm{T}}, \ u_2^{\mathrm{T}}, \ \ldots, \ u_N^{\mathrm{T}} \right]^{\mathrm{T}} \in \mathbb{R}^{Nm}, \tag{5}$$

$$G(x) = \begin{bmatrix} g(x_1) & & & \\ & g(x_2) & & \\ & & \ddots & \\ & & & g(x_N) \end{bmatrix} \in \mathbb{R}^{Nn\times Nm}. \tag{6}$$

Then the dynamics of the multi-agent system (1) can be expressed more compactly as

$$x(k+1) = (I_N \otimes A)x(k) + G(x(k))u(k). \tag{7}$$

The consensus errors (2) and their dynamics can be rewritten as

$$e(k) = (L \otimes I_n)x(k), \tag{8}$$

and

$$e(k+1) = (I_N \otimes A)e(k) + (L \otimes I_n)G(x(k))u(k), \tag{9}$$

respectively.

Assumption 1. The communication topology is described by a connected undirected graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V}$ is the indexed set of agents in the system and $\mathcal{E}$ represents the set of communication links among the agents.

Remark 1. Under Assumption 1, consensus is achieved if and only if all the elements in $e(k)$ approach zero as k goes to infinity. Since the Laplacian matrix L is symmetric, there exists some orthogonal matrix $T \in \mathbb{R}^{N\times N}$ such that

$$T^{\mathrm{T}}LT = \begin{bmatrix} 0 & & & \\ & \lambda_2 & & \\ & & \ddots & \\ & & & \lambda_N \end{bmatrix} \triangleq J_L.$$

By Lemma 1, we know that L is positive semi-definite and