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Research article

Relay tracking control for second-order multi-agent systems with damaged agents ☆

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ABSTRACT

This paper investigates a situation where smart agents capable of sensory and mobility are deployed to monitor a designated area. A preset number of agents start tracking when a target intrudes this area. Some of the tracking agents are possible to be out of order over the tracking course. Thus, we propose a cooperative relay tracking strategy to ensure the successful tracking with existence of damaged agents. Relay means that, when a tracking agent quits tracking due to malfunction, one of the near deployed agents replaces it to continue the tracking task. This results in jump of tracking errors and dynamic switching of topology of the multi-agent system. Switched system technique is employed to solve this specific problem. Finally, the effectiveness of proposed tracking strategy and validity of the theoretical results are verified by conducting a numerical simulation.

1. Introduction

During the past decades, multi-agent systems [1–5] are developed and have been widely used in the applications of mobile robots [6], unmanned vehicles [7–9] and spacecraft systems [10]. Tracking problem of a target is a typical issue of multi-agent systems and has drawn much attention of researchers [11–15]. In most of the existing literatures, the topologies of multi-agent systems are supposed to be fixed topologies [16] or switching topologies [12,13,17,18]. The tracking control of multi-agent systems under the effect of three kinds of partial mixed impulses is investigated in [19,20]. However, in these papers, the results achieved are based on the assumption that the tracking agents are fixed and replacement of tracking agents are not considered.

To bridge this gap in the literature, the authors in [21] propose a relay pursuit scheme to capture a maneuvering target on a plane, monitored by a group of agents. In [21], the agents are assumed to be first-order systems and the target is tracked by a single pursuer without cooperation. As mentioned in the work of [21], in some situations, the capture is successful only if the pursuers cooperate with each other. In that sense, relay pursuit strategies are viewed as an intermediate option offering a simpler alternative for a multi-agent tracking problem involving multiple agents, which is considered a hard problem to deal with [22]. Another possible extension is to consider the case that agents are described by more realistic kinematics instead of integral dynamics.

Motivated by the above discussion, we consider a more complicated scenario where a certain area is monitored by a large number of mobile agents, described by nonlinear dynamics, when targets move into this area, a preset number of mobile agents begin tracking the targets. We have achieved some results on cooperative relay tracking for first-order multi-agent systems [23,24]. In these two papers, the monitored domain is divided into many Voronoi cells with the assistance of knowledge of Voronoi diagrams. Then, when a target enters a new Voronoi cell, the Voronoi site agent replaces one of the original tracking agents. Furthermore, the overall stability conditions for a tracking system being unstable over part of time intervals are obtained in [24]. The stability conditions in [23,24] are established on the basis that the norm of tracking errors $\| \mathcal{E}(t)\|^2$ decrease at every switching time t_k, i.e., $\| \mathcal{E}(t_k^+)\|^2 \le \mu \| \mathcal{E}(t_k^-)\|^2$, $0 < \mu < 1$. The condition $0 < \mu < 1$ is guaranteed by the switching law that one of the original tracking agents is replaced by the Voronoi site agent when the target enters its Voronoi cell.

However, the condition $0 < \mu < 1$ is not easy to be always satisfied in practical applications such as protecting sensitive areas against offensive intrusion [25] and occasionally tracking targets in a typical military, environmental, or habitat monitoring applications [26]. In practical applications, along with the moving of target, one or some of the tracking agents may be damaged due to different causes, for example, the tracking agents could be attacked, out of power or in disorder. When one or some of the tracking agents are damaged, the equal number of the other deployed agents are

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supposed to join the tracking. Under this circumstance, the tracking error is likely to jump up at the switching times, i.e., $\mu > 1$, differing from the basis condition $0 < \mu < 1$ in [23,24]. Therefore, the theorems achieved in [23,24] are inapplicable for this situation. The replacement of damaged tracking agents triggers the change of dynamic tracking topology and possibly causes jump up of the tracking error. This has not been studied in the existing literature, to the best of our knowledge.

The practical situation addressed above is given to elaborate the motivation of this paper. Through solving the raised issue, some contributions and innovations are made and summarized below. Firstly, compared with the previous works [23,24], a more general case of tracking problem is explored and the practical problem is modelled in mathematical form, precisely describing the switch of tracking agents and arise of jump errors. Secondly, a new selection rule of the relay agent considering moving direction of the target is proposed for the second-order multi-agent systems. Finally, the stability for the relay tracking system with jump errors at switching instants ($\mu > 0$) is analyzed.

The following notations will be used throughout this paper. $\mathbb{N}$ is used to denote the set of nonnegative integers. $\mathbb{R}^m$ denotes the set of all $m-$ dimensional real column vectors. For a real symmetric positive semi-definite matrix P, $\lambda_i(P)$ denotes its $i-$ th eigenvalue. $\lambda_{max}(P)$ and $\lambda_{min}(P)$ represent the maximum and minimum eigenvalue of P, respectively. $\|\cdot\|$ stands for the Euclidean norm. $f_{\mathcal{K}}$ and $f_{\mathcal{KL}}$ belong to class $\mathcal{K}$ and $\mathcal{KL}$ functions, respectively.

2. Related preliminaries

In the view of graph theory, each agent can be treated as a node. Then the communication topology of tracking agents and the target can be treated as a dynamic graph. A weighted graph $G := \{\mathcal{N}, \mathcal{E}, \mathcal{A}\}$ is denoted by a node set $\mathcal{N} = \{1, 2, \dots, N_f\}$, an edge set $\mathcal{E} \subseteq \mathcal{N} \times \mathcal{N}$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N_f \times N_f}$ with nonnegative elements. An ordered edge of G is denoted by (i, j), representing that agent i is able to send information to agent j. If the graph is undirected, the edges (i,j) and (j,i) in $\mathcal{E}$ are considered to be the same. Throughout this paper, graph G is assumed to be undirected. The nodes within the communication range of node i are called the set of neighbours of node i, which is denoted by $\mathcal{N}_i = \{j | j \in \mathcal{N}, (j, i) \in \mathcal{E}\}$. When $j \notin \mathcal{N}_i$, it is said node j is beyond the communication range of node i, $a_{ij} = a_{ji} = 0$, otherwise $a_{ij} = a_{ji} > 0$. If $a_{ii} \neq 0$, we say that node has self-loop. In this paper, it is assumed that no self-loop exists. b_i decides whether agent i is able to communicate with the target or not. $b_i = 0$ means agent i cannot get the information of target, otherwise $b_i > 0$. Denote $\mathcal{B} = diag\{b_1, b_2, \dots, b_{N_f}\}$.

The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{N_f \times N_f}$ of graph G is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A}$, where

$$\mathcal{D} = diag\left\{ \sum_{j \in N_1} a_{1j}, \sum_{j \in N_2} a_{2j}, \dots, \sum_{j \in N_{N_f}} a_{N_f j} \right\}.$$

In this paper, the communication topology is no longer fixed as the tracking agents relay during the whole course. Thus, the Laplacian matrix is rewritten as $\mathcal{L}_{\sigma(t)}$, where $\sigma(t) = k$, $t \in [t_k, t_{k+1})$, $k \in \mathbb{N}$.

Denote the communication graph by $\mathcal{G} = \{G(0), G(1), G(2), \dots, G(N_s)\}$, where N_s is the total switching number and $G(k) = \{\mathcal{N}_{G(k)}, \mathcal{E}_{G(k)}, \mathcal{A}_{G(k)}\}$, $k \in \mathbb{N}$ is the communication graph. Denote the topology graph at time t as $\mathcal{G}_t$, then $\mathcal{G}_t = G(k)$ when $\sigma(t) = k$. Therefore, the topologies are piecewise constant between the event times $t_0, t_1, \dots, t_k$.

Let $N(t_0, t)$ denote the switching times during the time interval $[t_0, t)$. It satisfies the average dwell time property (1).

$$N(t_0, t) \le N_0 + \frac{t - t_0}{\tau_a}, \tag{1}$$

in which N_0 is a positive number and $\tau_a > 0$ represents the average dwell time.

The following definition is employed to obtain the main results.

Definition 1. [27] Let $x(t)$ be a solution of $\dot{x}(t) = f(t, x(t))$, $t \ge t_0$, $x(t_0) = x_0$, $t_0 \ge 0$ and $V(t, x(t))$ be a given function. Denote $D^+V(t, x(t))$ as the upper right-hand Dini derivative of $V(t, x(t))$, i.e.

$$D^+V(t, x(t)) = \limsup_{h \to 0^+} \frac{V(t + h, x(t + h)) - V(t, x(t))}{h}.$$

3. Problem formulation and relay tracking algorithm

3.1. Problem formulation

The kinematic equation of the i-th agent deployed on the monitored area is described as

$$\begin{aligned} &\dot{x}_i(t) = v_i(t) \\ &\dot{v}_i(t) = f(t, x_i(t), v_i(t)) + u_i(t), \quad t \ge t_0 \\ &x_i(t_0) = x_{i_0}, \quad v_i(t_0) = v_{i_0}, \quad t_0 \ge 0, \end{aligned} \tag{2}$$

where $x_i(t) \in \mathbb{R}^2$ is position state of the i-th agent, $v_i(t) \in \mathbb{R}^2$ is velocity state of the i-th agent, $f(t, x_i(t), v_i(t))$ is a nonlinear function, $u_i(t) \in \mathbb{R}^2$ is the control input of the i-th agent. $i = 1, 2, \dots N_f$ and N_f is the number of tracking agents.

The agents are expected to monitor this area and track any intruded target. We consider the following kinematic equations of the target.

$$\begin{aligned} &\dot{x}_{\mathbf{t}}(t) = v_{\mathbf{t}}(t) \\ &\dot{v}_{\mathbf{t}}(t) = f(t, x_{\mathbf{t}}(t), v_{\mathbf{t}}(t)), \quad t \ge t_0 \\ &x_{\mathbf{t}}(t_0) = x_{\mathbf{t}_0}, \quad v_{\mathbf{t}}(t_0) = v_{\mathbf{t}_0}, \quad t_0 \ge 0 \end{aligned} \tag{3}$$

where $x_{\mathbf{t}}(t) \in \mathbb{R}^2$ and $v_{\mathbf{t}}(t) \in \mathbb{R}^2$ are position and velocity state of the target, respectively. $f(t, x_{\mathbf{t}}(t), v_{\mathbf{t}}(t))$ denotes the nonlinear dynamics of the target. In this paper, we assume $f(\cdot)$ satisfies the following constraint.

Assumption 1. There exist finite constants $l_x \ge 0$, $l_v \ge 0$ such that $\| f(t, x_i, v_i) - f(t, x_{\mathbf{t}}, v_{\mathbf{t}})\| \le l_x \| x_i - x_{\mathbf{t}} \| + l_v \| v_i - v_{\mathbf{t}} \|$.

Assumption 1 is a Lipschitz-type condition, satisfied by many well-known systems including Lorenz system, Chen system, Lü system, Chua's circuit, and so on.

Definition 2. For second-order multi-agent tracking problems, not only the position but also the velocity of tracking agents are required to be consensus with the target. Then, it is said the tracking agents successfully track the target if the following conditions satisfy.

$$\begin{aligned} &\lim_{t \to \infty} \| x_i(t) - x_{\mathbf{t}}(t)\| = 0, \\ &\lim_{t \to \infty} \| v_i(t) - v_{\mathbf{t}}(t)\| = 0, \quad i = 1, 2, \dots N_f. \end{aligned} \tag{4}$$

It should be noted that for the first-order multi-agent systems, success of tracking and capture share the same meaning, requiring $\lim_{t \to \infty} \| x_i(t) - x_{\mathbf{t}}(t)\| = 0$. However, for the second-order multi-agent systems, the definitions of tracking and capture are different. $\lim_{t \to \infty} \| x_i(t) - x_{\mathbf{t}}(t)\| = 0$ guarantees the success of capture without any requirement on the velocities of agents. Whereas