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Research article

Scaled consensus seeking in multiple non-identical linear autonomous agents

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ABSTRACT

Scaled consensus problem is studied for a heterogeneous multi-agent system composed of non-identical stable linear agents, and a leader-following scaled consensus algorithm is designed. By using frequency-domain analysis, consensus conditions are obtained for the agents without communication delay under undirected and directed topologies, respectively. Moreover, consensus criteria are also gained for the agents suffering from communication delay under directed topology. Simulation examples show the correctness of theoretical results.

1. Introduction

As a fundamental phenomenon in flocking, swarming, and coupled synchronization, consensus behavior has attracted more and more attention of researchers in the control theory field in recent years. Consensus problem requires several autonomous agents to reach a common agreement on their states, and has broad engineering applications, e.g., coordination control of unmanned aerial and ground vehicles, smart grid, wireless sensor networks, etc.

Current research results on consensus problem have been mainly focused on homogeneous multi-agent systems, of which the agent's dynamics are identical [1–10]. With the help of algebraic graph theory [1], matrix theory [2–6], frequency-domain analysis [7–9], and Lyapunov functions [10], various consensus algorithms have been designed and analyzed for the agents with fixed topology or switching topologies.

Over the past few years, furthermore, a lot of research effort has also been put into the consensus analysis and synthesis of heterogeneous multi-agent system composed of the agents with distinct dynamics. As a simple heterogeneous multi-agent system, mixed system with multi-class agents has attracted many researchers' interests, such as the system of first-order agents and second-order agents [11,12] and the system of Euler-Lagrange agents and double-integrator agents [13,14]. For the heterogeneous linear multi-agent systems, state-feedback and output-feedback consensus algorithms were proposed by using the internal model principle [16,17] and the harmonic control [18]. To solve the consensus problem of heterogeneous linear multi-agent system with diverse communication delays, Lee and Spong [15] used the spectral radius theorem to obtain the delay-independent consensus conditions. Meanwhile, Muenz [19] and Tian and Zhang [20] adopted the consensus algorithms with devisable self delays distinct from the corresponding communication delays, and provided the consensus criteria based on frequency-domain analysis. In addition, Tian and Zhang [20] designed an adaptive adjustment algorithm in order to adjust the self delays on-line if the communication delays were unknown. Liu and Liu [21] proposed three adaptive consensus algorithms for the heterogeneous first-order time-invariant multi-agent systems, and obtained the consensus conditions for the agents under fixed topology without and with time delays respectively. Specially, non-identical disturbances also lead to the heterogeneity of multi-agent systems [22]. Based on Lyapunov functions, Kim et al. [22] adopted a general consensus algorithm and analyzed the robust consensus convergence condition For the heterogeneous first-order time-varying multi-agent systems with distinct disturbances.

Treated as the generalized consensus problem, scaled consensus problem defined in [23] means that the network components' scalar states reach the assigned proportions rather than a common value, and has attracted some researchers' interests for its engineering applications, such as compartmental mass-action systems, closed queueing networks, and water distribution systems [23]. In addition to usual consensus problem, group consensus problem [24], wherein the agents form several sub-groups reach corresponding agreement values respectively, is also a special case of scaled consensus problem. With a connected topology, Roy [23] proved that the first-order multi-agent systems not only achieved a stationary scaled consensus asymptotically, but also could track on the stable manifold. For the first-order multi-agent systems, Meng and Jia [25] studied the scaled consensus problem with time-varying scaled ratios, and demonstrated that the agents converged to the scaled consensus asymptotically under jointly-connected switching

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topologies. By constructing Lyapunov-Krasovskii functionals, Aghbolagh et al. [26] considered the scaled consensus of first-order multi-agent systems with time-varying delay, and gained the consensus conditions in the form of linear matrix inequalities, which were used to calculate the allowable delay value.

In this paper, we consider the scaled consensus problem of a class of heterogeneous multi-agent systems with the agents modeled by single-input and single output linear stable dynamics, and an adaptive scaled consensus algorithm is constructed in the leader-following structure. Based on frequency-domain analysis, firstly, we obtain the consensus conditions for the multi-agent systems without communication delay under undirected and directed topologies, respectively. For the multi-agent systems with identical communication delay, besides, the delay-independent and delay-dependent consensus conditions are obtained, respectively, for the system under directed topology.

2. Problem description

2.1. Non-identical agents and topology

Consider a class of heterogeneous multi-agent system composed of N non-identical single-input single-output linear agents given by

$$\begin{aligned}\dot{x}_i(t) &= A_i x_i(t) + B_i(u_i(t) + n_i),\\ y_i(t) &= c x_i(t),\ i = 1, \dots, N,\end{aligned} \tag{1}$$

$$A_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\alpha_{i0} & -\alpha_{i1} & -\alpha_{i2} & \cdots & -\alpha_{in-1}\end{bmatrix},\ B_i = \begin{bmatrix}0\\0\\\vdots\\0\\b_i\end{bmatrix},$$

$$c = \begin{bmatrix}1 & 0 & 0 & \cdots & 0\end{bmatrix}.$$

where $x_i(t) \in R^n$, $y_i(t) \in R$ and $u_i(t) \in R$ are the state, the output and the input of agent i, respectively, $n_i \in R$ is the constant disturbance of each agent i, $A_i \in R^{n\times n}$, $B_i \in R^n$, $c \in R^{1\times n}$, $b_i \in R$, and $\alpha_{ik} \in R$, $k = 0, \dots, n-1$. Correspondingly, the frequency-domain description of (1) is formulated as

$$Y_i(s) = g_i(s)(U_i(s) + N_i(s)),\ i = 1, \dots, N, \tag{2}$$

where $Y_i(s)$, $U_i(s)$ and $N_i(s)$ are the Laplace transforms of $y_i(t)$, $u_i(t)$ and n_i respectively, and $g_i(s)$ is the transfer function. Since the agents' dynamics (1) are formulated as a controllable canonical form, we get

$$\begin{aligned} g_i(s) &= \frac{b_i}{d_i(s)} \\ &= \frac{b_i}{s^n + \alpha_{in-1}s^{n-1} + \cdots + \alpha_{i1}s + \alpha_{i0}}.\end{aligned} \tag{3}$$

Assumption 1. The system matrix A_i, $i = 1, \dots, N$ is Hurwitz, i.e., $d_i(s)$, $i = 1, \dots, N$ is Hurwitz.

Evidently, A_i is non-singular with Assumption 1 guaranteeing that all the eigenvalues of A_i are non-zero.

Assumption 2. Without loss of generality, b_i is assumed to be positive, i.e., $b_i > 0$.

Definition 1. The multi-agent network (1) achieves *Scaled Consensus* asymptotically, if

$$\lim_{t\to\infty}(r_i y_i(t) - r_j y_j(t)) = 0,\ i, j = 1, 2, \dots, N, \tag{4}$$

where the ratios $r_i \in R$, $i = 1, \dots, N$ are assumed to be non-zero.

If $r_i = r_j$, $\forall\, i, j \in \{1, \dots, N\}$, the output scaled consensus in Definition 1 becomes an output synchronization. Hence, the output scaled consensus problem (4) is a generalized output synchronization problem.

Usually, the interconnection topology of multi-agent system (1) is described as an N-order weighted digraph $G = (V, E, \mathcal{A})$, which is composed of a set of vertices $V = \{1, \dots, N\}$, a set of edges $E \subseteq V \times V$ and a weighted adjacency matrix $\mathcal{A} = [a_{ij}] \in R^{N\times N}$ with $a_{ij} \ge 0$. In the diagraph G, a directed edge from the node i to the node j is denoted by $e_{ij} = (i, j) \in E$, and it is assumed that $a_{ij} > 0 \Leftrightarrow e_{ij} \in E$. Moreover, we assume $a_{ii} = 0$ for all $i \in V$. Besides, the digraph G is undirected graph or bidirectional if $e_{ij} \Leftrightarrow e_{ji}$, and the digraph is symmetric if $a_{ij} = a_{ji}$. The set of node i's neighbors is denoted by $N_i = \{j \in V: (i, j) \in E\}$. The Laplacian matrix of the weighted digraph G is defined as $L = D - \mathcal{A} = [l_{ij}] \in R^{n\times n}$, where $D = \mathrm{diag}\{\sum_{j=1}^{N} a_{ij},\ i \in V\}$ is the degree matrix.

In the digraph, if there is a path from one node i to another node j, then j is said to be *reachable* from i, otherwise, j is said to be not reachable from i. If a node is reachable from every other node in the digraph, then we say it *globally reachable*. A globally reachable node is precisely the degree of connectedness required of the digraph. An undirected graph is connected if it has a globally reachable node.

2.2. Design of scaled consensus algorithm

In this paper, we focus on the leader-following coordination control, which has been extensively studied for multi-agent systems [27–31]. For the agents (1), a usual scaled consensus algorithm is given by

$$\xi_i(t) = \frac{1}{r_i}\left(\sum_{j\in N_i} a_{ij}(r_j y_j(t) - r_i y_i(t)) + p_i(y_0 - r_i y_i(t))\right),\ i \in V, \tag{5}$$

where the non-zero constants r_i, $i \in V$ denotes the ratios of agents' states, N_i denotes the neighbors of agent i, $a_{ij} > 0$ is the adjacency element of $\mathcal{A}$ in the digraph $G = (V, E, \mathcal{A})$, y_0 is the static leader's state, and p_i denotes the linking weight from agent i to the leader. Assume that $p_i > 0$ if there is a directed edge from agent i to the leader; otherwise, p_i=0. In the rest of this paper, the notation $P = \mathrm{diag}\{p_i,\ i \in V\}$ is used. Apparently, the algorithm (1) is similar to that in [25].

Taking $u_i(t) = \kappa\xi_i(t)$ with $\kappa > 0$, we get the closed-loop form of multi-agent system (1) as follows

$$\dot{x}_i(t) = A_i x_i(t) + B_i(\kappa\xi_i(t) + n_i),\ i \in V. \tag{6}$$

Remark 1. With the interconnection topology that has a globally reachable node, it is easily proved that the multi-agent system (6) converges to an asymptotic scaled consensus, only if

$$r_i c A_i^{-1} B_i n_i = y_0,\ \forall\ i \in V. \tag{7}$$

Condition (7) can be obtained by assuming that the agents converge to the scaled consensus asymptotically, i.e., $\lim_{t\to\infty} r_i y_i(t) = y_0$.

Evidently, the condition (7) is dependent on the parameters strictly, i.e., the agents cannot achieve the asymptotic scaled consensus when the condition (7) is not satisfied. Moreover, (7) also implies that the agents (1) without disturbances cannot reach the asymptotic scaled consensus with the algorithm (5).

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