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Research article

Mean square consensus of leader-following multi-agent systems with measurement noises and time delays

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ABSTRACT

This paper investigates the mean square consensus problem of dynamical networks of leader-following multi-agent systems with measurement noises and time-varying delays. We consider that the fixed undirected communication topologies are connected. A neighbor-based tracking algorithm together with distributed estimators are presented. Using tools of algebraic graph theory and the Gronwall-Bellman-Halanay type inequality, we establish sufficient conditions to reach consensus in mean square sense via the proposed consensus protocols. Finally, a numerical simulation is provided to demonstrate the effectiveness of the obtained theoretical result.

1. Introduction

In recent years, consensus problems of multi-agent systems have been a hot research topic which attracted considerable attention in multidisciplinary research areas. When a group of agents is seeking agreement upon a certain quantity of interest, which might be attitude, position, velocity, voltage, and so on, consensus problems naturally arise. The theoretical framework to solve the consensus problems were proposed by Olfati-Saber and Murray [1]. There are numerous studies of deterministic multi-agent systems [2–5]. Many researchers concerned about the consensus of first-order dynamics in the earlier research. However, second-order systems have more important practical significance. The convergence of second-order multi-agent systems [3,4] relies not only on the information exchange topologies but also on the coupling strength between the derivatives of velocities. The case of second-order multi-agent system was studied in [5], which studied the consensus problem for a class of general second-order multi-agent systems with communication delay. In [6], this paper described a distributed coordination scheme with local information exchange for second-order systems, which extended first-order protocols from the literature. In [7], the second-order consensus problem for multi-agent systems with nonlinear dynamics and directed topologies was considered.

However, it is often difficult or impossible for a networked agent to obtain timely and accurate information of its neighbors due to environment uncertainties and communication delays. Moreover, link failures and formation reconfiguration make it necessary to research consensus problems for networks with these constraints taking into account. Dealing with the distributed consensus problems under measurement noises is not trivial, because the designed consensus protocol for each agent relies on its neighbor' states which are corrupted by noises and delays. For time-delayed systems, modeled by delayed differential equations, there are some effective methods to deal with convergence and stability problems such as Lyapunov-based [8,9], Lyapunov-Krasovskii functionals [10,11] or Lyapunov-Razumikhin functions [12–14], and so on. In [15], Wang et al. investigated adaptive synchronization of weighted complex dynamical networks with coupling time-varying delays. Adaptive controllers were designed for nodes of the controlled network, which have strong robustness against asymmetric coupling matrix, time-varying weights, delays, and noise. Based on the Kalman-Consensus filter and the flocking algorithm, Ref. [16] investigated the distributed estimation and control for mobile sensor networks with coupling delays. By applying an effective cascading Lyapunov method and matrix theory, stability analysis was carried out. Ref. [17] studied leader-following consensus problem of multi-agent systems with mixed delays and uncertain parameters. By utilizing the adaptive pinning intermittent control idea, a novel distributed control protocol was designed based only on local intermittent information, some novel

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criteria were derived in matrix inequalities form by resorting to the generalized Halanay inequality. Formation-containment control problems for multi-agent systems with second-order dynamics and time-varying delays were studied in [18]. Consensus control problems with measurement noises have been studied in [19–21]. Huang [22] proposed a stochastic approximation type algorithm for discrete-time consensus control and introduced a decreasing consensus gain to attenuate the measurement noise. The work of [22] was extended to continuous time system by [23], and necessary and sufficient conditions were given on the consensus gains to achieve asymptotic unbiased mean square average consensus. Wang [24] studied the containment consensus problem of first-order and second-order dynamics in a noisy communication environment, respectively. Sufficient and necessary conditions were obtained to achieve mean square consensus, which were relatively weaker conditions.

In addition, a more challenging problem in distributed cooperative control is the leader-following consensus, that is, the coordinated tracking problem. There were many fruitful results on consensus tracking of multi-agent systems under different scenarios [25–28]. Ref. [25] was concerned with the distributed $H\infty$ consensus of leader-follower multi-agent systems with aperiodic sampling interval and switching topologies. With help of the Lyapunov stability theory, a sufficient condition for the existence of mode-dependent controller was established and it guaranteed the exponential stability of tracking error system. A leader-following consensus problem of a group of autonomous agents with time-varying coupling delays was considered in [26], in which Lyapunov-Razumikhin function was employed along with the analysis of linear matrix inequalities. Ref. [27] studied robust consensus tracking problem for a class of second-order multi-agent dynamic systems. The distributed consensus protocols were designed to enable global asymptotic consensus tracking. In [28], by using tools from Lyapunov stability analysis and M-matrix theory, the distributed consensus tracking problem was discussed for high-order linear multi-agent systems. Robust finite-time consensus problems in leader-following multi-agent directed networks with second-order nonlinear dynamics were considered in [29]. In [30], a neighbor-based tracking protocol was designed for a leader-follower multi-agent system with measurement noises. In [31], the problem of leader-following output consensus of a linear discrete-time multi-agent system with input saturation and external disturbances was investigated. To deal with the output consensus of multi-agent systems, low-gain state feedback technique and output regulation theory were applied.

Motivated by these above observations, this paper is devoted to investigate the problem of mean square consensus of leader-following multi-agent systems with measurement noises and time delays. The main contributions of this paper can be summarized as follows:

(1) We consider a consensus problem with an active leader with an underlying dynamics. Here, the velocity of an active leader cannot be measured, and each agent only gets the measured information of the leader position once there is a connection between them. Our model considered in this paper is more consistent with the practical engineering.
(2) To estimate the velocity of the leader, a neighbor-based controller together with a neighbor-based state-estimation law is designed for each agent. Both the measurement noises and time-varying communication time-delays are taken into account in the presented protocol.
(3) The velocity decomposition technique and a distributed estimation algorithm for the velocity of the active leader are proposed to deal with tracking consensus of leader-following multi-agent systems in a noisy environment. A Gronwall-Bellman-Halanay type inequality is employed, which plays an essential role in proving the consensus results.
(4) The sufficient criterion are obtained to guarantee that the leader-following multi-agent systems reach tracking consensus in mean square, which are easily to verify.

The rest of paper is organized as follows. In Section 2, some basic definitions in graph theory and some preliminaries are given. The consensus protocol is proposed for stochastic leader-following multi-agent systems with measurement noises and time delays in Section 3. The detailed proofs for the main results are given by using Lyapunov function method. Then, numerical simulations are provided in Section 4. The paper is concluded by Section 5.

Notations: Let $\mathbb{R}$ denotes the real space; $\mathbb{R}^n$ denotes the n–dimensional Euclidean space; I_n denote an n dimensional identity matrix; $\|\cdot\|$ stands for the norm for both vectors and matrices, at the same time, we denote $\|\varphi\|_\tau = \sup_{\theta\in[-\tau,0]}\|\varphi(\theta)\|$ for a function $\varphi \in C([-\tau, 0], \mathbb{R}^n)$; 1_n and 0_n denote a column vector with all ones and zeros, respectively. A^T denotes the transpose of the matrix A; and Tr (A) is its trace. Let $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of the matrix A, respectively. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}, \mathcal{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\geq 0}$ which satisfies that the filtration contains all $\mathcal{P}$-null sets and is right continuous. Let $\mathfrak{L}^2_{\mathcal{F}_0}([-\tau, 0], R^n)$, the family of all $\mathcal{F}_0$-measurable $PC([-\tau, 0], R^n)$-valued random variables $\phi = \{\phi(s): -\tau \leq s \leq 0\}$ such that $\sup_{-\tau\leq s\leq 0}\mathbb{E}\|\phi(s)\|^2 < \infty$, where $\mathbb{E}(\cdot)$ represents the mathematic expectation of corresponding variable.

2. Preliminaries

Graph theory [32] is the basic theory to analyze the interconnection topologies. An undirected graph is denoted as $G = (v, \varepsilon, A)$, which consists of a set of nodes $v = \{v_1, v_2, \ldots, v_n\}$, an edge set $\varepsilon \subseteq v \times v$, and an adjacency matrix $A = [a_{ij}] \in R^{n\times n}$. Here, the node indexes belong to the finite index set $I = \{1, 2, \ldots, n\}$. If an edge of G is denoted as $e_{ij} = (v_i, v_j) \in \varepsilon$, it means node v_j can receive the information from node v_i, where v_i is the parent node and v_j is the child node. A graph is connected if there is a path between any two distinct nodes. Let $N_i = \{v_j: (v_i, v_j) \in \varepsilon\}$ denotes the set of neighbor of node. A nonnegative weighted matrix A is called an adjacent matrix, which specifies the interconnection topology of network. If the edge of G satisfies $(v_i, v_j) \in \varepsilon$, the element of adjacency matrix is defined as $a_{ii} = 0$ and $a_{ij} = 1$ for all $i \neq j$. For an undirected graph, A is a symmetric and every undirected graph is balanced. Moreover, a diagonal matrix $D = \mathrm{diag}(d_1, d_2, \ldots, d_n)$ is called input degree matrix whose diagonal element is defined as $d_i = \sum_{j\neq i} a_{ij}$ for $i \in I$. The matrix $L = D - A$ is called the Laplacian matrix of the weighted graph. According to the definition, the row sum of the Laplacian matrix is zero and the column vector 1_n is the right eigenvector corresponding to a zero eigenvalue. If the graph is balanced, there is also a left eigenvector 1_n corresponding to a zero eigenvalue except the right one. Next we assume node 0 is the leader of the graph $\bar{G}$, which is used to describe the multi-agent system topology with one active leader agent and follower agents. Clearly, $\bar{G}$ is connected if and only if at least one agent in each component of G is connected to the leader. Take the diagonal matrix $B = diag\{a_{10}, \ldots, a_{n0}\} \in \mathbb{R}^{n\times n}$ as a leader adjacency matrix, where $a_{i0} = 1$ if the follower agent ν_i can receive the information from the leader ν_0, and $a_{i0} = 0$ otherwise. Let a new matrix $H = L + B$ denotes the connectivity of the leader graph. Note that $L1_n = 0$, so it follows that $H1_n = B1_n$.