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Research article

Distributed adaptive asymptotically consensus tracking control of uncertain Euler-Lagrange systems under directed graph condition[☆]

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ABSTRACT

In this paper, a backstepping based distributed adaptive control scheme is proposed for multiple uncertain Euler-Lagrange systems under directed graph condition. The common desired trajectory is allowed totally unknown by part of the subsystems and the linearly parameterized trajectory model assumed in currently available results is no longer needed. To compensate the effects due to unknown trajectory information, a smooth function of consensus errors and certain positive integrable functions are introduced in designing virtual control inputs. Besides, to overcome the difficulty of completely counteracting the coupling terms of distributed consensus errors and parameter estimation errors in the presence of asymmetric Laplacian matrix, extra information transmission of local parameter estimates are introduced among linked subsystem and adaptive gain technique is adopted to generate distributed torque inputs. It is shown that with the proposed distributed adaptive control scheme, global uniform boundedness of all the closed-loop signals and asymptotically output consensus tracking can be achieved.

1. Introduction

Due to its wide potential applications in a diversity of fields, distributed consensus control of multi-agent systems has received significant attention from various research communities over the past decades. Normally, it aims at achieving an agreement for the states or the outputs of different network-connected dynamic subsystems, by designing a local controller for each subsystem based on only information collected within its neighboring area. According to whether the final consensus values are predetermined, this control issue can be classified into leaderless consensus control [1–3] and leader-following consensus control [4–8]. Note that in most of existing results in leader-following consensus control, the desired trajectories are set by the behaviors of specific leaders with similar dynamics to the followers and zero/known inputs. If more general cases are considered with common desired trajectories given by certain time-varying functions, such issue is often referred to as consensus tracking control in the literature; see for instance [9–12].

Compared with traditional tracking control of single systems, the main difference of distributed consensus tracking control of multi-agent systems lies in a relaxed condition of centralized information such that the common desired trajectory is not necessarily known by all the subsystems. However, such difference constitutes the major difficulty in controller design and consensus analysis. To solve this problem, several effective distributed control protocols have been proposed. In [4,6,7], partial knowledge of the reference trajectories are assumed available to all of the subsystems. Distributed observers are then designed in the subsystems which cannot fully access the reference trajectory to estimate its remaining uncertainties. In [8], to generate the local control law of each subsystem, the information of both states and control signals of its neighbors need be collected. Thus perfect consensus tracking can be achieved though the reference trajectory is totally unknown by some subsystems. However, such design of mutually dependent inputs may bring new challenges during implementation if they are generated without a prescribed priority [13]. Alternative solutions to asymptotically consensus tracking are provided in [14] and [9], by introducing signum functions of local consensus errors in the proposed distributed control approaches.

Moreover, it is worth mentioning that most of the aforementioned results are developed by assuming that the subsystem

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models are precisely known. However, intrinsic subsystem uncertainties invoked by imperfect modeling are actually unavoidable in practice. Though adaptive control is a promising tool to handle parametric uncertainties with robustness to structural and environmental uncertainties, the results on distributed adaptive consensus control of uncertain multi-agent systems are still limited especially when the information transmission condition is represented by directed graphs. This is mainly due to the difficulty in constructing distributed adaptive laws with only locally available information [15] if the symmetric property of Laplacian matrix is no longer applicable. In [17], consensus tracking of first-order multi-agent systems with unknown nonlinear dynamics and disturbances under directed topology is investigated. By incorporating neural network and robust control techniques, semi-global uniform ultimate boundedness of consensus errors is ensured if the local control gains are chosen to be sufficiently large. The results are extended to systems with higher-order multi-agent systems in [12] and [18]. In [19], it is assumed that the reference trajectory is linearly parameterized and the basis function vectors are known by all the subsystems. Then distributed adaptive control algorithm is presented for achieving perfect consensus tracking of first-order multi-agent systems with unknown parameters. The results are extended in [20] to solve adaptive finite-time consensus problem of uncertain higher-order distributed agents. Nevertheless, the considered information transmission topology is undirected and the proposed distributed adaptive laws can only be implementable with extra information transmission of local neighbourhood consensus errors among the linked subsystems. In [10], based on similar assumption on linearly parameterized reference trajectories, asymptotically output consensus tracking of uncertain nonlinear multi-agent systems is firstly achieved by developing distributed adaptive controllers under directed graph condition. Similar problem to [10] is handled in [11] by adopting dynamic surface design approach and semi-globally uniformly ultimately bounded consensus tracking errors are finally obtained.

Recently, a new smooth function based distributed adaptive tracking control scheme is proposed in [21] for nonlinear multi-agent systems with unknown parameters and uncertain disturbances. Based on the scheme, asymptotically output consensus tracking can be achieved without the assumption on linearly parameterized reference trajectory and known basis function. Nevertheless, the information transmission condition is assumed to be represented by an undirected graph. In this paper, we focus on extending the results in [21] to the case of directed graph, by considering multiple Euler-Lagrange systems with parametric uncertainties. For better identifying the contributions of this paper, the main differences between our results and currently available representative references are summarized as follows.

i) In this paper, we suppose the information transmission status among different subsystems is represented by a directed graph. Thus the control protocols and stability/consensus analysis presented in [6,7,19–21] by employing the graph symmetry property are not applicable to solve our problem.
ii) In contrast to [6,7,10,19], the assumptions of linearly parameterized reference signals and the corresponding basis function vectors being known by all subsystems are no longer needed. In this paper, it is assumed that the desired trajectories on m different dimensions (i.e. $p_{r,k}(t)$, $1\le k\le m$, in later problem formulation) are known exactly by only part of the subsystems. For the remaining subsystems, it is only known that each $|\dot{p}_{r,k}(t)|$ is upper bounded by an arbitrarily unknown constant F_k.
iii) Rather than achieving only bounded consensus errors in the existing results on distributed adaptive control under directed graph conditions, such as [11,12,17,18], asymptotically output consensus tracking can be guaranteed by introducing extra information transmission of local parameter estimates of F_k among linked subsystems.

The remainder of the paper is organized as follows. In Section 2, the considered output consensus tracking problem for a group of N Euler-Lagrange subsystems are firstly formulated. Then the design of distributed adaptive controllers and stability/consensus analysis are presented in Sections 3 and 4, respectively. Simulation results are provided in Section 5 followed by a conclusion drawn in Section 6.

2. Problem formulation

2.1. System model

In this paper, we consider a group of N Euler-Lagrange subsystems modeled as follows.

$$M_i(p_i)\ddot{p}_i + C_i(p_i, \dot{p}_i)\dot{p}_i + g_i(p_i) = \tau_i,\ 1 \le i \le N \tag{1}$$

where $p_i = [p_{i1}, \dots, p_{im}]^T \in \Re^m$ and $\tau_i \in \Re^m$ are the generalized coordinate and control input torque of the ith subsystem, respectively. $M_i(p_i) \in \Re^{m\times m}$ is the symmetric positive definite inertia matrix, $C_i(p_i, \dot{p}_i)$ is the Coriolis and centrifugal matrix, $g_i(p_i)$ is the vector of gravitational force.

2.2. Information transmission condition for the N subsystems

We suppose that the information transmission condition for the group of N subsystems can be represented by a fixed, directed graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{1, \dots, N\}$ denotes the set of indexes corresponding to each subsystem, $\mathcal{E} \subseteq \mathcal{V}\times\mathcal{V}$ is the set of edges between two distinct subsystems. An edge $(i, j) \in \mathcal{E}$ indicates that subsystem j can obtain information of subsystem i, but not necessarily vice versa [22]. In this case, subsystem i is called a neighbor of subsystem j. We denote the set of neighbors for subsystem i as $\mathcal{N}_i$, i.e. $\mathcal{N}_i \triangleq \{j \in \mathcal{V}: (j, i) \in \mathcal{E}\}$. Note that self edges (i,i) are not allowed, thus $(i, i) \notin \mathcal{E}$ and $i \notin \mathcal{N}_i$. The connectivity matrix $A = [a_{ij}] \in \Re^{N\times N}$ is defined such that $a_{ij} = 1$ if $(j, i) \in \mathcal{E}$ and $a_{ij} = 0$ if $(j, i) \notin \mathcal{E}$. Clearly, the diagonal elements a_{ii} of A satisfies that $a_{ii} = 0$. We introduce an in-degree matrix $\triangle$ such that $\triangle = \mathrm{diag}(\triangle_i) \in \Re^{N\times N}$ with $\triangle_i = \sum_{j\in\mathcal{N}_i} a_{ij}$ being the ith row sum of A. Then, the Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L} = \triangle - A$. A node is balanced if its in-degree equals its out-degree, i.e. $\sum_{j=1}^{N} a_{ij} = \sum_{j=1}^{N} a_{ji}$. A directed graph is balanced if all its nodes are balanced.

In a directed graph, a sequence of successive edges $\{(i, k), (k, m), (m, l), (l, j)\} \subseteq \mathcal{E}$ denotes a directed path from node i to node j. A directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is strongly connected, if there is a directed path from any node $i \in \mathcal{V}$ to any other nodes $j \in \mathcal{V}$. For a digraph, its underlying graph is the graph obtained by replacing all its directed edges with undirected edges. The digraph is weakly connected if its underlying graph is connected.

2.3. Control objective

In this paper, the output consensus tracking objective is to design distributed adaptive controllers τ_i for each subsystem i based on only locally available information such that

i) all the signals in the closed-loop system are globally uniformly bounded;
ii) the outputs of all the N subsystems can track a common desired trajectory $p_r(t) \in \Re^m$ asymptotically while maintaining

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