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Research article

Robust output synchronization of heterogeneous nonlinear agents in uncertain networks

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ABSTRACT

This paper investigates the global robust output synchronization problem for a class of nonlinear multi-agent systems. In the considered setup, the controlled agents are heterogeneous and with both dynamic and parametric uncertainties, the controllers are incapable of exchanging their internal states with the neighbors, and the communication network among agents is defined by an uncertain simple digraph. The problem is pursued via nonlinear output regulation theory and internal model based design. For each agent, the input-driven filter and the internal model compose the controller, and the decentralized dynamic output feedback control law is derived by using backstepping method and the modified dynamic high-gain technique. The theoretical result is applied to output synchronization problem for uncertain network of Lorenz-type agents.

1. Introduction

Controlling a group of agents and rendering their outputs to common reference trajectory is one fundamental objective in many control problems for multi-agent systems. In recent years, the researches on output synchronization problem have been expanded from linear into nonlinear scenario. For different kinds of nonlinear multi-agent systems under various settings of network topologies, certain notable concepts and design methodologies have been proposed to achieve output synchronization, for instance, the concepts of passivity and dissipativity [5,18], the cyclic-small-gain theorem [17], the pinning control method [9,30,35], and the references therein. Besides, other design issues regarding the engineering applications have also been discussed, such as the input saturation [20–22], the unknown control direction [16], the switching network topology and the bounded synchronization, [18,19,25–27], to name only a few.

It is noticeable that the classical output regulation theory and the internal model principle also exhibit their great potentials in accomplishing such kind of problems. In the output regulation theory, the reference trajectory and external disturbance are governed by an exogenous dynamics known as the exosystem, which plays the same role as the virtual leader in the leader-following format. For nonlinear multi-agent systems, the so-called cooperative or distributed output regulation have been studied for agents of several standard nonlinear forms, e.g., the normal form [19,23], the output feedback form [6–8,31,32,34], the lower triangular form [11,24,36], etc. Furthermore, in the leaderless format, the internal model based design also provides certain feasibility to synchronize nonlinear multi-agent systems, c.f. [13,29]. The results therein exploit the importance of embedding the internal model as part of the controller to achieve synchronization.

In this paper, the global robust output synchronization problem is considered for a class of nonlinear multi-agent systems where the heterogeneous agents are in output feedback form with different relative degrees, and the communication network is specified by an uncertain digraph. Provided that there exists a directed spanning tree with the virtual leader as the root, the dynamic output feedback controller comprising the input-driven filter and internal model can be designed, and the global robust output synchronization is achieved which admits the bounds of parametric uncertainties for both agents and network and the bound of external disturbances are not known a priori.

Our main contributions are two folds: 1) The control design is independent of any explicit or implicit quantitative information of the communication network among agents, i.e., since the network is completely unknown, quantitative information regarding the size of the corresponding digraph, the weights of the edges, so as the underlying adjacency matrix or the Laplacian matrix is not used to achieve the control law. 2) The control is of decentralized type so that it overcomes the restriction that no information exchange is allowed among controllers. Also, the design avoids the fully collaborative manner and simplifies most of the procedures to perform w.r.t. each agent separately. In turn, it provides certain feasibility to cope with heterogeneous agents with different relative degrees.

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The rest of the paper is organized as follows. In Section 2, we formulate the global robust output synchronization problem. In Section 3, the problem is pursued via nonlinear output regulation theory and the internal model based design. In Section 4, the theoretical result is applied to an output synchronization problem for Lorenz-type agents in uncertain networks. And the paper is concluded in Section 5.

Notations: Given a column vector x_i, $x_{i,j}$ denotes its j-th component. For column vectors $x_i \in \mathbb{R}^{n_i}$, $\mathrm{col}(x_1, \dots, x_n) \triangleq \left[x_1^\top, \dots, x_n^\top\right]^\top \in \mathbb{R}^{n_1+\cdots+n_n}$. In the similar manner, a collection of vector-valued functions f_i is denoted by $\mathrm{col}(f_1, \dots, f_n)$. $A \triangleq [a_{ij}] \in \mathbb{R}^{n\times m}$ denotes a matrix with elements a_{ij}. I_n denotes an identity matrix of size n. $D \triangleq \mathrm{diag}[d_1, \dots, d_n]$ denotes a diagonal matrix with d_i being its i-th diagonal element.

In the following context, any parameter that can be arbitrarily chosen is referred to as a free parameter. For brevity, the arguments of a function are omitted when no ambiguity occurs.

2. Problem formulation

Consider a group of heterogeneous nonlinear agents $\mathcal{S}_i$, $i = 1, \dots, n$,

$$\mathcal{S}_i: \quad \begin{aligned} \dot{z}_i &= f_i(z_i, y_i, v, w), \\ \dot{x}_{i,j} &= g_{i,j}(z_i, y_i, v, w) + x_{i,j+1}, \quad j = 1, \dots, r_i, \\ y_i &\triangleq x_{i,1}, \quad u_i \triangleq x_{i,r_i+1}. \end{aligned} \tag{1}$$

Agent $\mathcal{S}_i$ is in nonlinear output feedback form with relative degree r_i, $r_i \ge 1$. $\mathrm{col}(z_i, x_i)$ is the state, $z_i \in \mathbb{R}^{n_i}$, $n_i \ge 0$, $x_i = \mathrm{col}(x_{i,1}, \dots, x_{i,r_i}) \in \mathbb{R}^{r_i}$, where z_i represents the the dynamic uncertainty, i.e., the unmodeled dynamics. $u_i \in \mathbb{R}$ and $y_i \in \mathbb{R}$ are the control input and performance output, respectively. $w \in \mathbb{W}$ represents the parametric uncertainty, $v \in \mathbb{V}$ describes the external disturbance, where $\mathbb{W} \subset \mathbb{R}^{n_w}$ and $\mathbb{V} \subset \mathbb{R}^{n_v}$ are compact sets with unknown bounds.

Compared with the controlled agents $\mathcal{S}_i$, it is assumed that the reference trajectory y_0 is generated by the following autonomous agent,

$$\mathcal{S}_0: \quad \dot{v} = A_0 v, \quad y_0 = g_0(v, w). \tag{2}$$

where $\mathcal{S}_0$ is neutrally stable, i.e., all eigenvalues of A_0 are semi-simple with zero real parts. Consequently, $\mathcal{S}_0$ is forward complete and invariant w.r.t. any compact set $\mathbb{V}$, i.e., for any $v(0) \in \mathbb{V}$, it is certain that $v(t) \in \mathbb{V}$, $\forall\, t \in \mathbb{R}^+$.

It is also assumed that all functions in (1)–(2) are smooth and satisfying $f_i(0, 0, 0, w) = 0$, $g_{i,j}(0, 0, 0, w) = 0$ and $g_0(0, w) = 0$, thus the origin is the equilibrium point of $\mathcal{S}_i$.

For the given system $\{\mathcal{S}_0, \mathcal{S}_1, \dots, \mathcal{S}_n\}$, the underlying communication can be characterized by a weighted digraph $\bar{\mathcal{G}}$. If $\mathcal{S}_i$ receives information provided by $\mathcal{S}_j$, then there exists a directed edge $<j, i>$ with unknown weight $\bar{a}_{ij} > 0$, otherwise $\bar{a}_{ij} = 0$. Correspondingly, the weighted adjacency matrix of $\bar{\mathcal{G}}$ is defined by $\bar{\mathcal{A}} \triangleq [\bar{a}_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$, and the weighted Laplacian matrix is defined by $\bar{\mathcal{L}} \triangleq [\bar{l}_{ij}]$, where $\bar{l}_{ii} = \sum_{j=0}^{n} \bar{a}_{ij}$ for $i = 0, 1, \dots, n$, and $\bar{l}_{ij} = -\bar{a}_{ij}$, $i \ne j$. Based on $\bar{\mathcal{G}}$, the measurement output e_i^m and the tracking error e_i for agent $\mathcal{S}_i$ are defined as follows,

$$e_i^m = \sum_{j=0}^{n} \bar{a}_{ij}(y_i - y_j), \quad e_i = y_i - y_0, \quad i = 1, \dots, n. \tag{3}$$

Note that $\bar{\mathcal{L}}$ can be partitioned as $\bar{\mathcal{L}} = \left[\begin{array}{c|c} 0 & 0\cdots 0 \\ \hline \bar{l} & \mathcal{L} \end{array}\right]$, where $\mathcal{L} \triangleq [l_{i,j}] \in \mathbb{R}^{n\times n}$ and $\bar{l} = \mathrm{col}(\bar{l}_{10}, \dots, \bar{l}_{n0})$, so (3) can be alternatively written into the following compact form,

$$e^m = \mathcal{L}e, \quad e^m = \mathrm{col}(e_1^m, \dots, e_n^m), \quad e = \mathrm{col}(e_1, \dots, e_n). \tag{4}$$

The global robust output synchronization problem is formulated as follows: design the following dynamic output feedback control law for each controlled agent $\mathcal{S}_i$,

$$\dot{\chi}_i = \alpha_i(\chi_i, e_i^m), \quad u_i = \gamma_i(\chi_i, e_i^m), \tag{5}$$

such that for any initial conditions $\mathrm{col}\big(z_i(0), x_i(0)\big) \in \mathbb{R}^{n_i+r_i}$, any $v \in \mathbb{V}$, $w \in \mathbb{W}$, the states of the closed-loop system composed of (1) and (5) are bounded, and $\lim_{t\to+\infty} e_i = 0$, i.e., the performance output y_i tracks the reference trajectory y_0 asymptotically.

It is well comprehended that the network topology plays an essential role in achieving the aforementioned output synchronization problem. To make our statement self-sustained, we propose the following assumption.

Assumption 1. The digraph $\bar{\mathcal{G}}$ is simple[1] and contains a directed spanning tree with node 0 as the root. ∎

Remark 1. Assumption 1 implies that all controlled agents $\mathcal{S}_i$ are reachable from $\mathcal{S}_0$. It is quite a standard and necessary assumption that provides the "minimal connectedness" of the static network topology to synchronize all agents' outputs, see Remark 2.2, 3.2 in [34] for detailed discussions.

Notice that $\mathcal{L}$ in (4) is an uncertain matrix since the size and the weights a_{ij} are not known a priori. However, it is fully recognized that $\mathcal{L}$ is indeed a non-singular $\mathcal{M}$-matrix under Assumption 1. And for any non-singular $\mathcal{M}$-matrix, there always exists a positive diagonal matrix $D \triangleq \mathrm{diag}[d_1, \dots, d_n]$ such that

$$D\mathcal{L} + \mathcal{L}^\top D = Q, \tag{6}$$

for some positive definite matrix Q [1]. The existence of matrix D ensures that $(\delta D)\mathcal{L} + \mathcal{L}^\top(\delta D)$ is also a positive definite matrix for any constant $\delta > 0$. ∎

3. Output synchronization via output regulation theory

In this section, the aforementioned global robust output synchronization problem is pursued via nonlinear output regulation theory. Some basic assumptions regarding the output regulation theory are proposed first. Then the dynamic controller is designed which consists of the input-driven filter and internal model, and the output synchronization problem is converted into a decentralized stabilization problem. The relevant stabilization problem is further achieved via certain nonlinear control techniques.

3.1. Basic assumptions

Assumption 2. For each $\mathcal{S}_i$, there exists a smooth function $\mathbf{z}_i(v, w)$: $\mathcal{R}^{n_v} \times \mathcal{R}^{n_w} \mapsto \mathcal{R}^{n_i}$ satisfying $\mathbf{z}_i(0, w) = 0$, such that, for any $v \in \mathbb{V}$, $w \in \mathbb{W}$, $\frac{\partial \mathbf{z}_i(v,w)}{\partial v} A_0 v = f_i\big(\mathbf{z}_i(v, w), g_0(v, w), v, w\big)$. ∎

Assumption 2 provides partial solution of the regulator equation associated with (1) and (2), and the rest can be obtained as follows: denote $\mathbf{x}_{i,1}(v, w) = g_0(v, w)$, then $\mathbf{x}_{i,j+1}(v, w) = \frac{\partial \mathbf{x}_{i,j}(v,w)}{\partial v} A_0 v - g_{i,j}\big(\mathbf{z}_i(v, w), g_0(v, w), v, w\big)$, $j = 1, \dots, r_i$, $\mathbf{u}_i(v, w) = \mathbf{x}_{i,r_i+1}(v, w)$. Let $\mathbf{x}_i(v, w) = \mathrm{col}\big(\mathbf{x}_{i,1}(v, w), \dots, \mathbf{x}_{i,r_i}(v, w)\big)$, then $\mathrm{col}\big(\mathbf{z}_i(v, w), \mathbf{x}_i(v, w)\big)$ and $\mathbf{u}_i(v, w)$ are termed as the steady-state state and steady-state input of $\mathcal{S}_i$, respectively, and they characterize the steady-state

[1] When $\bar{\mathcal{G}}$ contains no self-loops or parallel directed edges, it is called simple.