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Research article

# Distributed synchronization of networked drive-response systems: A nonlinear fixed-time protocol

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ABSTRACT

The distributed synchronization of networked drive-response systems is investigated in this paper. A novel nonlinear protocol is proposed to ensure that the tracking errors converge to zeros in a fixed-time. By comparison with previous synchronization methods, the present method considers more practical conditions and the synchronization time is not dependent of arbitrary initial conditions but can be offline pre-assign according to the task assignment. Finally, the feasibility and validity of the presented protocol have been illustrated by a numerical simulation.



## 1. Introduction

The synchronization phenomenon has been known since the 17th century through the observation work of Christian Huygens on synchronization of two pendulum clocks. Synchronization as an important and interesting collective behavior of complex networks has been extensively studied. And many kinds of synchronization have been introduced, such as complete synchronization [1], projective synchronization [2], impulsive synchronization [3], cluster synchronization [4], and so on.

In sense, synchronization between two systems (with state space $x(t), y(t)$) means the asymptotic stability of the tracking errors, i.e., $|x(t) - y(t)| \to 0$ as $t \to \infty$. Up to now, many significant methods aiming at synchronization of drive-response systems can be found, including the linear feedback control [5], nonlinear feedback control [6], sliding mode control [7], adaptive control [8], to mention but a few [9–11].

However, most of above results only focus on a specific dynamical system and assume that all system parameters are same. From a practical point of view, more conditions should be considered such as system nonlinearity and communication delay, which are inherent vice. Communication delay naturally arises from a realistic consideration of traffic congestions and finite speed of information transmission between pairs of nodes, and it has a great impact on their collective behavior [12]. Wang and Qian [13] investigated local exponential synchronization of a general complex network with a single coupling delay. Using a combination of Riccati differential equation approach, Lyapunov $C$ Krasovskii functional, inequality techniques, the synchronization of non-autonomous drive-response systems with time-varying delay was investigated via delayed feedback control in [14].

On the other hand, convergence rate, as a significant performance index, is a hot research topic in control problem. To address this issue, fixed-time stability theory, first discovered in [15], has been received significant attention, which means better robustness and disturbance rejection properties [16]. Due to Polyakov [17], a sufficient condition for the fixed-time stability of nonlinear system was concluded. The fixed-time stability requires that the convergence time of the closed-loop system is uniformly bounded with respect to initial conditions, and this property is very promising in practice [18]. The fixed-time consensus control protocol was first designed for multi-agent systems with single integrator dynamics in [19]. Zuo [20] proposed a novel consensus protocol by using terminal sliding mode for single-integrator dynamics. Recently, this technology has been extended to

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second-order multi-agent systems [21] and high-order cases [22]. Tian and Zuo [22] proposed a fixed-time consensus proposal for high-order multi-agent system by designing a novel sliding manifold. More detail can be found in [23–25].

To the best of our knowledge, there has not been published literatures on the fixed-time synchronization of networked drive-response systems. As mentioned earlier, drive-response systems widely exist in practice and it is imperative to discuss the fixed-time synchronization for networked drive-response systems, which serves as the main motivation of our work. By constructed suitable Lyapunov function, sufficient conditions are deduced to guarantee the fixed-time synchronization of the networked drive-response systems. Additionally, the settling-time of fixed-time case is independent of the changing of agents initial states, for which, an accurate estimation can be preset by the design parameters of the protocol.

The rest of the paper is outlined as follows. Section 2 details the problem formulation and some preliminaries. The main result of our research is elaborately presented in Section 3. The numerical simulation is shown to verify the effectiveness of proposed method in Section 4. Finally, we conclude this paper in Section 5.

*Notations:* $\mathcal{R}^n$ denotes the $n$-dimensional Euclidean space; $()^T$ denotes the transpose of matrix or vector; $\otimes$ stands for Kronecker product; $diag()$ is diagonal matrix operation; $I_n$ represents the $n \times n$ identity matrix. For a vector $x = \left[x_1, x_2, \ldots, x_n\right]^T \in R^n$ and $p \in R$, define $x^p = \left[x_1^p, x_2^p, \ldots, x_n^p\right]^T$; For a given matrix $A \in \mathcal{R}^{n\times n}$, $\|A\| = \left(Tr\left(A^TA\right)\right)^{1/2}$ denotes the Euclidean norms operation; $\lambda_1(A), \lambda_n(A)$ denote the minimum and maximum eigenvalues of matrix $A$, respectively.

## 2. Problem description and preliminary

### 2.1. Algebraic graph theory

In this section, some basic concepts and lemmas on algebraic graph theory are introduced, which will be used in the later analysis. For more details, we refer the readers to [26]. For a system of $n$ agents, information can be transmitted between neighbors, so it is natural to describe the topology of the information flow by a weighted graph. Let $\mathcal{G} = \left\{\nu, \varepsilon, \mathcal{A}\right\}$ denotes a weighted graph, where $\nu = \left\{1, \ldots, n\right\}$ is the node set, $\varepsilon \subset \nu \times \nu$ represents the edge set, and $\mathcal{A} = \left[a_{ij}\right]$ is the weighted adjacency matrix of graph $\mathcal{G}$. Node $i$ denotes the $i$th agent, and the adjacency element $a_{ij}$ denotes the information interaction between the $i$th and the $j$th agent, i.e. $(i, j) \in \varepsilon \Leftrightarrow a_{ij} > 0$. Meanwhile, node $i$ and node $j$ are called neighbors, and accordingly, $\mathcal{N}_i = \left\{j\middle|(i, j) \in \varepsilon\right\}$. For any two nodes $i$ and $j$, if there at least exists one path between them, then $\mathcal{G}$ is called a connected graph. The graph $\mathcal{G}$ is simple if it has no self-loop or repeated edges.

Define the graph $\vec{\mathcal{G}}$ to describe the interconnection topology of a multi-agent system consisting of one leader and $n$ followers. Let $\mathcal{B} = diag\left(b_1, \ldots, b_n\right)$ be the leader adjacency matrix associated with graph $\mathcal{G}$, where $b_i > 0$ is a constant if the *ith* follower has access to the leader, $b_i = 0$, otherwise. The Laplacian $\vec{\mathcal{L}}$ of graph $\vec{\mathcal{G}}$ is defined as $\vec{l}_{ii} = \sum_{j=1}^n a_{ij} + b_i$ for $i = j, i, j \in \left\{1, \ldots, n\right\}$ and $\vec{l}_{ij} = -a_{ij}$ for $i \neq j, i, j \in \left\{1, \ldots, n\right\}$.

**Lemma 1** ([26]). *Assuming the graph $\vec{\mathcal{G}}$ is connected, the Laplacian matrix $\vec{\mathcal{L}}$ is symmetric and positive definite, and its n real eigenvalues can be arranged as an ascending order:*

$$0 < \lambda_1\left(\vec{\mathcal{L}}\right) \leqslant \lambda_2\left(\vec{\mathcal{L}}\right) \leqslant \cdots \leqslant \lambda_n\left(\vec{\mathcal{L}}\right) \leqslant 2\,\vec{l}_M$$

*where* $\vec{l}_M = \max_{1\leqslant i\leqslant n}\left\{\vec{l}_{ii}\right\}$.

In this paper, we only consider that the graph $\mathcal{G}$ is undirected and fixed ( i.e., $\mathcal{A}$ is time-invariant matrix ). It is the basics for different kinds of topologies and its application can be found in most multi-agent literatures [26].

### 2.2. Problem description and mathematical preliminaries

Consider the following nonlinear systems, as a leader labeled 0, which is usually called as drive system

$$\dot{x}_0(t) = Ax_0(t) + Bg\left(x_0(t)\right) + Cg\left(x_0(t-\tau)\right) \tag{1}$$

and a class of controlled followers systems, named response systems

$$\dot{x}_i(t) = Ax_i(t) + Bg\left(x_i(t)\right) + Cg\left(x_i\left(t-\tau(t)\right)\right) + u_i \tag{2}$$

where $x_i \in R^m$ are system state vectors; $g(\cdot)$: $R^m \to R^m$ are nonlinear vector functions; $A$, $B$ and $C$ are system matrices with proper dimensions; $\tau(t)$ is the system time-varying delay, where $0 \leqslant \tau(t) \leqslant \bar{\tau}$, $\dot{\tau}(t) \leqslant h < 1$, and $h$ is known constant. For simplicity, in the sequel the terms $g\left(x_i(t)\right), g\left(x_i\left(t-\tau(t)\right)\right)$ are replaced as $g_i, g_{i,\tau}$ in case there is no confusion.

To streamline the technical proof of our main result, the following definition, assumption and lemmas are needed.

**Definition 1.** Consider a non-Lipschitz system $\dot{x} = f(x)$ with $f(0) = 0$. The origin of the system is said to be globally fixed-time stable if it is globally uniformly finite-time stable and the settling-time function $T$ is globally bounded.

**Assumption 1.** For any $x, y \in R^m$, the function $g(\cdot)$ satisfies Lipschitz condition, that is, there exists a constant $l_1 > 0$ such that,

$$\|g(x) - g(y)\| \leqslant l_1\|x - y\| \tag{3}$$

**Lemma 2** ([27]). *Consider a scalar system*

$$\dot{y} = -\alpha y^{\frac{m}{r}} - \beta y^{\frac{p}{q}}, \quad y(0) = y_0 \tag{4}$$

*where $\alpha, \beta > 0$, $m$, $r$, $p$ and $q$ are both positive odd integers satisfying $m > r$, $q > p$ . Then, the system state $y$ will converge to equilibrium point within the settling-time $T$, where*

$$T = \frac{1}{\alpha}\frac{m}{m-r} + \frac{1}{\beta}\frac{q}{q-p} \tag{5}$$

If $\varepsilon \triangleq \left[q(m-r)\right]/\left[r(q-p)\right] \leqslant 1$, a less conservative estimation of the settling-time $T$ can be obtained instead as

$$T = \frac{q}{q-p}\left(\frac{1}{\sqrt{\alpha\beta}}\tan^{-1}\sqrt{\frac{\alpha}{\beta}} + \frac{1}{\alpha\varepsilon}\right)$$

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